THE MATHEMATICAL GAZETTE

EDITED BY
W. J. GREENSTREET, M.A.
WITH THE CO-OPERATION OF
F. S. MACAULAY, M.A., D.Sc.
AND
Prof. E. T. WHITTAKER, M.A., F.R.S.

LONDON
G. BELL & SONS, LTD., PORTUGAL STREET, KINGSWAY, W.C. 2.
AND BOMBAY

Vol. XII., No. 178. OCTOBER, 1925. 2s. 6d. Net.

CONTENTS.

PAGE

Frontispiece—W. W. Rouse Ball.

W. W. Rouse Ball, M.A. Prof. E. T. Whittaker, F.R.S., - - - 449

Note on Systems of Conics satisfying four Conditions treated by the Theory of Characteristics. The Rev. J. J. Milne, M.A., - - 455

The Principle of Similarity. W. J. Dobbs, M.A., - - - - - 461

Mixture of "Arabic" with "Roman" Numerals during the Time of Transition. E. M. Langley, M.A., - - - - - - - - - 468

Mathematical Notes (796-804). R. W. Genese, M.A.; N. M. Gibbins, M.A.; H. G. Green, M.A.; M. J. M. Hill, F.R.S.; J. A. Johnston, M.A.; Prof. E. H. Neville, M.A.; R. E. Newton, B.Sc.; Prof. Sommerville, B.Sc., - - - - - - - - - - - - 470

Reviews. N. J. Chignell, M.A.; E. T. Copson, M.A.; Prof. H. Hilton, D.Sc.; Miss H. P. Hudson, O.B.E., Sc.D.; F. C. Mustard, B.Sc.; Prof. E. H. Neville, M.A.; Prof. H. T. H. Piaggio, D.Sc.; A. Robson, M.A., - - - - - - - - - - - - - 479

Gleanings Far and Near (325-330), - - - - - - - - - - 454

The Pillory; Mathematical Association—Victoria Branch; The Geometry Report; The Library, - - - - - - - - - - 488

Books Received, Etc., - - - - - - - - - - - - - i

Intending members are requested to communicate with one of the Secretaries. The subscription to the Association is 15s. per annum, and is due on Jan. 1st. It includes the subscription to "The Mathematical Gazette."

Change of Address should be notified to a Secretary. If Copies of the "Gazette" fail for lack of such notification to reach a member, duplicate copies can be supplied only at the published price.

The Mathematical Association.

PUBLICATIONS.

"**The Mathematical Gazette**" is issued six times a year. The price per copy (to non-members) is usually 2s. 6d. net.

The following **Reports** have been issued by the Association:

(1) Reports on the Teaching of Elementary Mathematics, 1902-1908 (Geometry, Arithmetic and Algebra, Elementary Mechanics, Advanced School Mathematics, the Course required for Entrance Scholarships at the Universities, Mathematics in Preparatory Schools), 6d. net. [Out of print.]

(2) Revised Report on the Teaching of Elementary Algebra and Numerical Trigonometry (1911), 3d. net.

(3) Report on the Teaching of Mathematics in Preparatory Schools (1907), 3d. net.

(4) Report on the Correlation of Mathematical and Science Teaching, by a Joint Committee of the Mathematical Association and the Science Masters' Association (1909, reprinted 1917), 6d. net.

(5) A General Mathematical Syllabus for Non-Specialists in Public Schools (1913), 2d. net. [Out of print.]

(6) Catalogue of Current Mathematical Journals, etc., with the names of the Libraries in which they may be found (1913), 40 pp., 2s. 6d. net.

(7) Report on the Teaching of Elementary Mathematics in Girls' Schools (1916), 1s. net.

(8) Report on the Teaching of Mechanics (*The Mathematical Gazette*, No. 137, Dec. 1918), 1s. 6d. net.

(9) Report on the Teaching of Mathematics in Public and Secondary Schools (*The Mathematical Gazette*, No. 143, Dec. 1919), 2s. net.

(10) Reports on the Teaching of Arithmetic and Algebra, Elementary Mechanics and Advanced School Mathematics (1922), 1s. net. Reprinted from (1).

(11) Report on the Teaching of Mechanics in Girls' Schools (1923), 6d. net.

(12) Report on the Teaching of Geometry in Schools (1923), 2s. net.

(13) Report on the Teaching of Mathematics in Preparatory Schools (1924), 1s. net.

Such of these Reports as are not out of print may be obtained from Messrs. G. Bell & Sons, Ltd., or from the Secretaries.

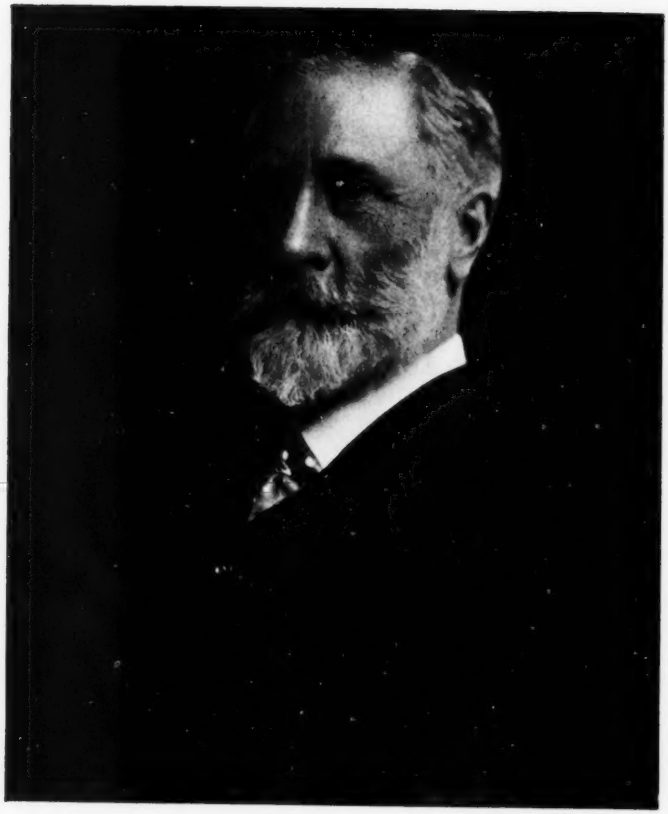


Photo. Russell, London.

W. W. ROUSE BALL, M.A.

THE

MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc., AND PROF. E. T. WHITTAKER, M.A., F.R.S.

LONDON:

G. BELL AND SONS, LTD., PORTUGAL STREET, KINGSWAY

AND BOMBAY.

VOL. XII. OCTOBER, 1925. No. 178.

Obituary.

W. W. ROUSE BALL.

MATHEMATICIANS all over the world have heard with regret of the death of one who was known to many of them as a man of singularly attractive personality, and to all of them as the author of scientific and historical works of great value and interest.

Walter William Rouse Ball was born on 14th August, 1850, the only son of Walter Frederick Ball, of Hampstead; and began his academic career at the age of seventeen, when he won an Entrance Exhibition to University College, London.

At this time it was not unusual for promising young mathematicians to take a degree at University College before going up to Cambridge. Todhunter and Routh had done it some years before, and the Senior Wranglers of the years immediately before and after Ball's year were both University College men. Of the small company of his fellow-students in London, several rose to distinction afterwards, notably the physicists W. E. Ayrton and J. A. Fleming, J. N. Keynes the logician, and the present Master of St. John's. The Professor of Mathematics was T. A. Hirst, a geometer who had been trained on the Continent, and who had succeeded de Morgan in the chair in 1867.

Concurrently with his mathematical course, Ball read Logic and Moral Philosophy under Professor Croom Robertson, and in 1869 graduated with Honours in that subject. To those who knew him only in the second half of his life, this early interest in philosophical studies seemed to have left but little residue. I never remember hearing him take an active part in philosophical discussion, or comment on philosophical books: and it is difficult to believe that the account of the Greek period in his *History of Mathematics* (long and valuable though it is) was the work of a mind fundamentally interested in philosophical questions.

In 1870 Ball went up to Trinity College, Cambridge, as a Minor Scholar, being elected in the following year to the foundation. The group of scholars of Trinity, among whom he now found himself, included the brothers Gerald Balfour (afterwards classical lecturer of the College and Chief Secretary for Ireland) and F. M. Balfour (afterwards Professor of Animal Morphology),

the brothers S. H. Butcher (afterwards President of the British Academy and M.P. for the University), and J. L. Butcher (Lord Danesfort), the classical scholars Verrall, Archer-Hind, and Hicks, Rendall (Headmaster of Charterhouse), Gow (the historian of Greek Mathematics), W. Cunningham the economist, F. W. Maitland (Downing Professor of Laws), A. B. Kempe (ecclesiastical lawyer and Treasurer of the Royal Society), and W. Leaf, the banker and Homeric scholar. The mathematical teaching (which in those days was entirely within the College) was provided by H. M. Taylor, W. D. Niven, J. W. L. Glaisher, and H. Lamb. Ball was regarded as one of the most promising men of his year, and indeed as the prospective Senior Wrangler: he was, however, placed Second in the Tripos of 1874. The Smith's Prize examination followed close on the Tripos, and Ball came out at the head of the list. In 1875 he was elected to a Trinity Fellowship.

His ambition now seems to have been towards a career in law. In 1876 he was called to the Bar by the Inner Temple, going the South-Eastern circuit. This, like the degree in Philosophy, is something of a puzzle: for he was a shy and reluctant speaker before any audience larger than a small committee. However, towards the end of the same year, W. K. Clifford, who had been appointed to the Chair of Applied Mathematics in University College, London, about the time Ball had gone up to Cambridge, broke down in health, and Ball was invited to act as his deputy. With the double experience of life as a practising barrister and as an academic teacher of mathematics, Ball now saw clearly where his real gifts and interests lay: and when in 1878 the opportunity came of returning to Trinity as lecturer, he had no hesitation in accepting it. For the next forty-seven years the tall bearded figure with the upright carriage was to be seen daily in the Great Court. He had found his life-work: and before long he began to put forth the works of scholarship on which his reputation rests.

The first of these, *The Origin and History of the Mathematical Tripos* (1880), was a pamphlet of only thirty-four pages, which, as its substance was incorporated in his later studies, need not be noticed specially here: but in 1888 appeared the *Short History of Mathematics*, which won an immediate and lasting popularity.

The causes of that popularity were not far to seek. The book may be compared in some respects with a historical work in a different field by another member of the College, Macaulay's *England*. In both cases, success was due in the first instance to the unparalleled ease, clarity and attractiveness of the style: the story was told round personalities, and was enlivened from time to time by deft touches of the picturesque, the dramatic, and the scandalous: and as Macaulay's work was designed primarily to glorify the Whigs, and in particular William of Orange, so it may perhaps be said that in some of Ball's presentation there was an undercurrent of glorification of Trinity, and in particular of Newton.

The task of the historian of science is, above all, to trace the origin and development of fruitful ideas: to reconstruct the intellectual outlook, the antecedent knowledge, the desires, the gropings, the intuitions, the connexions of thought, in the minds of the great investigators; and to exhibit the growth of conceptions and the elaboration of theories as they are transmitted from one mind to another.

In one part of his book—that devoted to the life and works of Newton—Ball comes very near to this ideal. A deep and penetrating study of all the literature of the Newtonian age had given him a complete mastery of the subject: and his analysis and solution of the difficult questions which it presents was a notable contribution to learning.

In dealing with other periods he was sometimes less successful. At the beginning of the book, in the account of Greek mathematics, he fails to conjure up the atmosphere and setting of Greek thought, or to indicate clearly the ruling

principles of the different schools and their relations to each other: and as a result, he leaves us with little more than a string of names, with anecdotes and theorems attached to them. In justice to Ball, it should be said that the two well-known (and in many respects excellent) Histories of Mathematics by American scholars, which have appeared since his, fail in this particular quite as deplorably as he did. The first really intelligible account in English of the origin of mathematics, so far as I know, was written not by a mathematician but by a classical scholar, Professor John Burnet, in his *Early Greek Philosophy*.

The other place where all Histories of Mathematics break down is the modern period. The development of the science has been continuous, and is proceeding as rapidly as ever to-day: several thousand research papers—most of them, it must be confessed, of no great merit—are published every year. To compass this vast literature, extended throughout the last century, is beyond human powers; and even if the task could be achieved by some superman, to give a proper account of it in two or three chapters of a moderate-sized book would be impossible. Professor David Eugene Smith cuts the knot by simply omitting all reference to the work of men now living, and giving only a hasty sketch of what has happened since 1830, confined almost entirely to personal history. Professor Cajori, on the other hand, makes an heroic effort, and devotes (in his second edition) over two hundred pages to the nineteenth and twentieth centuries. The labour of preparation must have been enormous; and the result is extraordinarily careful and accurate. I do not suppose that any one else could have done it better; but for all that, it is unsatisfactory: it is, in fact, scarcely readable; and one might hazard the guess that of the hundreds of names of modern mathematicians to be found in his colossal index, the majority will in a few years be totally and deservedly forgotten.

Between these extreme policies, Ball steered with his native tact and with some measure of success. While the fifty pages which he devotes to the nineteenth century in his third edition cannot be taken very seriously, they display the outlines of development pretty fairly: and his tendency to gossip always saves him from becoming hopelessly dull.

To write a general history of mathematics, from Thales to Einstein, is a task for which perhaps no one is ever likely to be perfectly qualified. Ball produced a sound and good book which has given delight and instruction to thousands, and we may leave it at that.

His next literary project was remarkably well chosen. It was original in conception, perfectly commensurate with his powers, and such as none but he could have written: it may, I think, be described as his most interesting and most valuable work. I refer to the *History of the Study of Mathematics at Cambridge*, which was published in 1889.

Of this it is difficult to say anything that is not unstinted praise. From the first page to the last it is as fascinating as a good novel; one could almost believe that Ball had lived in Cambridge continuously since the thirteenth century and remembered everybody that had taught there. The Jacobean mathematicians, Newton and his disciples, the eighteenth-century professors who never lectured, the revival of Cambridge mathematics in the days of the Analytical Society, the history of the Tripos, were topics on which he knew everything that was to be known, and told exactly what was worth telling.

About this time the Syndics of the University Press were publishing a series of schoolbooks, under the name of the *Pitt Press Series*. H. M. Taylor was invited to prepare an edition of Euclid, which was published in 1889-95, and is still in my opinion the best text-book of school geometry; and Ball was invited to prepare an *Algebra*, which appeared in 1890. It is clearly written, and thoughtfully warns the young student against innumerable pitfalls—rather an unexpected feature, since Ball cannot have had much experience of elementary teaching; but it follows the traditional exposition closely, and is not specially

remarkable for outlook, originality, or rigour. I do not know to what extent it came into use.

During these years Ball's pen was very active: in 1892 he produced *Mathematical Recreations and Problems*, a work so well-known that to describe it here is superfluous. The subject was congenial to him, and he had tremendous resources in this field; the idea of a book of mathematical recreations was not new, but he extended it in many fresh directions. The best parts—and they are very good—are those dealing with arithmetical and geometrical pastimes such as magic squares, mazes, and the knight's path on a chess-board. In the more subtle argumentative topics, such as the Paradoxes of Zeno and Hyper-Space, he is less successful, from the defect of philosophical quality in his mind. The three famous problems of antiquity—the duplication of the cube, the trisection of the angle, and the quadrature of the circle—he introduces by legends and discusses as isolated puzzles: this is hardly adequate, for to the Greeks the three problems were involved in a serious and far-reaching logical programme of research into the nature of things. But against these and all other criticisms may be set the fact that the copy which now lies before me is coming to pieces from constant use.

The *Essay on Newton's Principia*, which appeared in the following year (1893), was an excellent piece of work, the fruit of long-continued original investigations. It is greatly to be regretted that Ball never found time to carry out his intention of bringing out a critical edition of the *Principia* itself. Some shorter Newtonian studies, on the *Enumeratio linearum tertii ordinis*, and on a fragment relating to centripetal forces, preserved in the Portsmouth collection, were published* in the *Proceedings of the London Mathematical Society*. About this time he wrote several other papers in the mathematical journals: one, of date 1891, on *a hypothesis relating to the nature of the ether and gravity*.† As is well known, the wave-theory of light, in the form in which it was presented in the nineteenth century, postulated a medium or "ether," filling all space and having elastic properties similar to those of an ordinary solid body. It was a standing difficulty in the way of this theory that the hypothetical medium does not reveal its existence in any of the other ways we should expect, *e.g.* by resisting the motion of bodies immersed in it. Ball's hypothesis, which was intended to get over this difficulty, was that the ether occupies a three-dimensional space like our own, but very slightly displaced from it in the fourth dimension; so that every particle of matter (being, so to speak, a point of contact between the two spaces) is in contact with the ether, and can influence other particles of matter by means of vibrations propagated through the ether, although the ether itself, as a whole, is not in our space at all, and therefore cannot be perceived directly by us. One wonders that this hypothesis was never taken up by the spiritualists. Perhaps they never heard of it.

In the same volume‡ he discussed Mersenne's numbers, *i.e.* the values of p which make $2^p - 1$ a prime number. Soon after appeared a note on the resolution of numbers of a certain form into factors,§ and then a short paper on a new method of constructing magic squares. To a Swedish journal which had been established specially for papers on the History of Mathematics he contributed a note,‖ *On the Use of a Single Symbol to denote the Incommensurable Number* 3·14159....

Meanwhile, important events had taken place in his life. In 1893 he succeeded Glaisher as Tutor of Trinity: and he and Mrs. Ball (to whom he had been married in 1885) soon won the reputation of being the best tutor and tutor's wife that had ever been. They built a house not far beyond the Trinity Fellows' Garden, in the new western suburb which had begun to rise when the compulsory celibacy of Fellows was abolished; and here they welcomed and

---

* *Proc. L.M.S.* **22** (1891), p. 104; **23** (1892), p. 226. † *Mess. of Math.* **21** (1891), p. 20.
‡ *Ibid.* **21** (1891), p. 34. § *Ibid.* **22** (1892), p. 82. ‖ *Bibl. Math.* 1894, p. 206.

entertained an endless succession of pupils. Their charm as host and hostess was beyond all power of description: both of them were, I think, shy and reserved by nature, but shyness was swallowed up in goodness of heart.

With the influence of his new position, Ball set himself to foster the corporate life of the College. He was the moving spirit in the institution of a common-room for undergraduates, of a reference-library and reading-room for them, of breakfasts and luncheons in Hall, the acquisition of a new playing-field, and the formation of the Trinity Field Club. All these were highly successful except the breakfasts in Hall, which were subjected to a rule requiring the meal to be begun before 9.0 a.m., and consequently after a short trial were given up.

With the same motive he began to issue at the beginning of each academic year a little pamphlet of *Trinity Notes*, which was given to everyone on his "side." The topics were such as would be likely to interest the men and strengthen College feeling: the right age for coming up, the careers of those who were going down, the College clubs, inter-College and inter-University contests, compulsory Greek in the Little-go, and plans for making a sub-way to Whewell's Court.

In 1898 he became Senior Tutor and Chairman of the College Education Committee, and was now in a position to do more than ever for his cherished hope of making Trinity the chief centre of mathematical discovery. The young men who were elected to Fellowships just about the end of the century—Bertrand Russell, Barnes, Hardy, Jeans, and myself—were tremendously keen on research, and we had the warmest encouragement from the older Fellows, particularly Forsyth and Whitehead. On the administrative side, Ball saw to it that we were given every chance. A spirit was thus created which carried all before it, and reacted powerfully on the teaching of the undergraduates, many of whom have since become men of distinction; among the scholars of Trinity in the first decade of the present century who have since become Fellows of the Royal Society may be mentioned Eddington, Nicholson, Littlewood, Mercer, Watson, G. I. Taylor, Darwin, Chapman, and Proudman.

In addition to his College duties, Ball served the University as a member of the Financial Board, Auditor of the Chest, and (from 1905) representative of the University on the Borough Council. He was also a Governor of Westminster School and of the Perse School, and Treasurer of the University Boat Club.

During the strenuous years of Tutorship, he had little time to write books; and when he returned to literary work, it was as historian of the College. The *History of Trinity College, Cambridge*, the *History of the First Trinity Boat Club*, and the *Records of Admission to Trinity College*, 1546-1900, were followed in 1918 by a collection of his magazine articles and lectures to undergraduate societies, which was published under the title *Cambridge Papers*. About half of them are concerned with Trinity—the Chapel, the Library, the Portraits, the Statues, the Plate, the Auditors, the Scholars, and everything else that he knew so well—and the rest with studies of Newton and the University. Appreciation of their charm is by no means confined to those who know Cambridge.

Soon after the end of the War he founded a society for the promotion of conjuring in the University. Last year in writing to him I referred to it and enquired about its origin; the following is taken from his reply:

"Some five years ago I founded a society of undergraduates interested in conjuring and such like shows: if members can conjure so much the better, but that is not essential. I took as our symbol the re-entrant pentacle, which, as you may perhaps know, was sometimes used as a sign by magicians in the middle ages and probably also in classical times. The Society prospers, and at the end of an academic year numbers something like 100 members. Nearly four years ago we received the unexpected compliment of being asked to give a show in London before the leading professional conjurers. The invitation

was embarrassing, but could not be declined: however, in fact, we gave a really good show.

"I was the more inclined to introduce a pentacle by the fact that, according to the historians, it was the symbol selected by Pythagoras as the badge of his school. This seems to be true, and hence to mathematicians it should have interest."

On one occasion Ball himself gave to the members an exhibition of string figures—a subject on which he had written a short pamphlet.

The most sensational of the Club's public displays was given last Lent Term. A woman was sawn in two, before an audience who saw her head and feet continuously from the time she came on to the platform to the time when, having been reconstituted, she walked away again. It was rumoured that in the preliminary rehearsals she had once been injured actually.

Ball's interest in games of all kinds, which made him to the end of his life such an entertaining talker to undergraduates, was once the subject of a burlesque in the *Granta*. It was at the time when the success of bridge and ping-pong (then newly introduced) had led to an outburst of invention. A game, of a decidedly lively character, was advertised and described under the name of "Rous-ball."

Another of his interests was making a collection of portraits of mathematicians, past and present: probably the most extensive that has ever been formed. In 1914 he kindly allowed it to be exhibited at the Napier Tercentenary Celebration in Edinburgh; and a catalogue of the subjects will be found printed in the *Handbook to the Napier Exhibition*. Seven volumes are devoted to the General Collection, one to mathematicians of special excellence, and one to professors and University lecturers in mathematics at Cambridge. He attended the Napier Celebration, at which many who were specially interested in the History of Mathematics were gathered, and was received with marked respect. Afterwards he instituted a medal, commemorative of the discoveries of Napier, to be awarded annually in the University of Edinburgh to the most distinguished graduand in Mathematics.

The death of Mrs. Ball in December 1919 was a great blow to him, from which perhaps he never really recovered; but outwardly he seemed well and active. Last September he was pleased to meet again many of his old pupils, who had come up to Cambridge to celebrate the six hundredth anniversary of the founding of Michaelhouse (from which Trinity ultimately developed). I noticed that he walked with a stick, and understood that he had had some trouble affecting the heart, but in mind he seemed as bright and alert as ever.

He passed away at Elmside on Saturday, 4th April: the funeral service, attended by many from far and near, was held in the Chapel of Trinity on Wednesday, 8th April.

E. T. WHITTAKER.

GLEANINGS FAR AND NEAR.

325. Newton and Poetry.—A friend once said to him, "Sir Isaac, what is your opinion of poetry?" His answer was: "I'll tell you that of Barrow;—he said that poetry was a kind of ingenious nonsense." (Singer's edition of Joseph Spence's *Anecdotes*: Supplemental Anecdotes, Second Memorandum Book, 1756.) Spence gives as his authority John Robartes, Earl of Radnor, F.R.S. (1732). Prof. E. Bensly (*N. and Q.*, April 11, 1925) is reminded by Barrow's opinion of the Cambridge mathematician who "after the perusal of *Paradise Lost* delivered himself of the criticism that the author had proved nothing, a story to which Bishop Lightfoot presumably alluded when he remarked in a University sermon that there were some men to whom the noblest work of the imagination seemed nothing but a tissue of unproven statements."

NOTE ON SYSTEMS OF CONICS SATISFYING FOUR CONDITIONS TREATED BY THE THEORY OF CHARACTERISTICS.

By the Rev. J. J. Milne, M.A.

1. The systematic treatment of these systems by this method has been strangely neglected hitherto by English writers, the only text-book, as far as I am aware, which deals at all with the subject being Salmon's *Conics*, sixth edition, 1879, pp. 389, 390, where he states that the properties have been studied by de Jonquières, Chasles, Zeuthen, and Cayley, and that references to the original memoirs will be found in Cayley's memoir (*Phil. Trans.*, 1867, p. 75). This I have not had the opportunity of consulting. In this article I have followed the Danish mathematician, H. G. Zeuthen, who selected the subject for his Doctor's Thesis of 1865, which was translated and published in the *Nouvelles Annales de Mathématiques* in 1866, p. 241, with the title "A New Method of Determining the Characteristics of Systems of Conics."

2. The properties of the systems are made to depend on two numbers which Chasles called the characteristics of the system, and are:

(1) The number μ of the conics of the system which pass through a given point; and

(2) The number ν which touch a given straight line, and the symbol (μ, ν) is taken to represent such a system.

3. It will be seen that the values of μ, ν in any system are intimately connected with the number of what are called degenerate conics (D.C.) which the system contains. These D.C.'s are of two kinds, and we will briefly consider their peculiar properties.

If AA', BB' are the axes of a conic, and we suppose one of them, BB', to be indefinitely diminished, the conic reduces to a double line terminated by the points A, A', which we will call a D.C. of the 1st kind, and if this conic is cut by any st. line L, the points of intersection on L will be indefinitely near and consecutive, but the points of intersection on the conic, although indefinitely near, are not consecutive, and the conic does not touch the line.

To obtain the 2nd kind we suppose a double cone to be cut by a plane passing through the vertex, and we obtain a D.C. consisting of two straight lines, their point of intersection being a double point. In the ordinary conic, if from a point S we draw two tangents SP, SQ, and suppose one of them, SQ, whilst remaining a tangent, to move into coincidence with SP, the ultimate point of intersection S will be a point on the curve. In the case of a D.C. of the 2nd kind, if O is the intersection of the lines, and S any external point, the line SO passes through two indefinitely near but not consecutive points on the D.C., and if any other straight line be drawn through S and move to coincidence with SO, the point S will not be a point on the D.C.

4. The theory of characteristics is based on the Principle of Correspondence enunciated by Chasles in the *Comptes Rendus*, 24th December, 1855, viz.:

Given two pencils of rays, vertices A and B, such that to each ray of the first pencil A correspond β rays of the second pencil B, and to each ray of the pencil B correspond α rays of the pencil A, the ray AB not being a self-corresponding ray, the locus of the points of intersection of corresponding rays of the two pencils is a curve of order $\alpha+\beta$, *i.e.* a curve which cuts any straight line in $\alpha+\beta$ points.

The proof given by Chasles was algebraical, but it is easy to give a geometrical one. Let AL be one of the rays of the pencil A. This will be cut by the corresponding rays of B in β points. Corresponding to the ray BA

of the pencil B we have α rays of the pencil A. Consequently there are $\alpha+\beta$ points of the locus on AL, A being a multiple point of order α.

5. From this we at once deduce two Lemmas.

LEMMA I. On a straight line L given two series of points x and u such that to a point x of the first correspond α points of the second, and to each point u correspond β points x; the number of points x which coincide with the corresponding points u is $\alpha+\beta$.

LEMMA II. Given two pencils of rays X and U having the same vertex P, if to a ray PX correspond α rays PU, and to a ray PU correspond β rays PX, the number of rays PX which coincide with the corresponding rays PU is $\alpha+\beta$.

6. We will now give a theorem showing the relation between the values of the characteristics μ, ν, and the number of D.C.'s in any system.

THEOREM. *In every system of conics* (μ, ν) *the number of D.C.'s of the 1st kind is* $2\mu-\nu$, *and the number of the 2nd kind is* $2\nu-\mu$.

Let x be any point taken on any line L. Through x there pass μ conics of the system, and each of these conics meets L in a point u. $\therefore$ there are μ points u which correspond to the point x, and similarly there are μ points x which correspond to each point u. $\therefore$ by Lemma I. there are 2μ points x which coincide with the corresponding point u.

Now the number of conics which touch L is by def. ν.

$\therefore$ the difference $2\mu-\nu$ must refer to the number of D.C.'s of the 1st kind which meet the line L in two coincident points without touching it.

To prove the second part of the enunciation, through a point S draw a tangent SX to a conic of the system. Through S we can draw a second tangent SU, and as by hypothesis there are ν conics which touch the line SX, there will be ν lines SU corresponding to the line SX, and similarly to each line SU will correspond ν lines SX.

$\therefore$ by Lemma II. the pencil $S(X, U)$ will have 2ν double rays which will correspond to the number of conics which pass through the point S together with the D.C.'s of the 2nd kind for which SX coincides with SU without the conic passing through S. Now μ is by def. the number of conics which pass through the point S; $\therefore$ $2\nu-\mu$ is the number of D.C's of the 2nd kind, or a multiple of it.

7. Here we may point out that our object being to find the values of μ and ν, the number of D.C.'s in any system can be determined by inspection, and from them the characteristics can be calculated, provided we first find out how many D.C.'s of each kind exist. It will be seen that some of these D.C.'s are to be counted more than once, which rather complicates matters, but the number of repetitions required can be inferred from rules which are given in Art. 11.

8. In any system (μ, ν) let λ, π denote the number of D.C.'s of the 1st and 2nd kind respectively.

Then
$$\lambda = 2\mu - \nu,$$
$$\pi = 2\nu - \mu;$$
$$\therefore \mu = \tfrac{1}{3}(2\lambda+\pi), \quad \ldots\ldots(1)$$
$$\nu = \tfrac{1}{3}(\lambda+2\pi). \quad \ldots\ldots(2)$$

9. In what follows we will confine our attention for the present to the five elementary conic systems where the four data are: (A) 4 points; (B) 3 points 1 tangent; (C) 2 points 2 tangents; (D) 1 point 3 tangents; (E) 4 tangents.

In each of these cases the number of D.C.'s can be at once determined from a figure, but, as already mentioned in Art. 7, a difficulty arises owing to its being necessary to ascertain how many times each of the D.C.'s which we have found is to be counted in the numbers λ and π. The following is Zeuthen's method of dealing with this difficulty.

10. Denote by p the condition of passing through a given point, and by l the condition of touching a given straight line. The five elementary systems can then be written:

$$(p_1, p_2, p_3, p_4) \equiv (\mu_1, \nu_1),$$

$$(p_1, p_2, p_3, l) \equiv (\mu_2, \nu_2), \text{ etc.},$$

and (λ_1, π_1), (λ_2, π_2), ... will denote the values of λ and π relative to the different systems.

We assume that only one conic can be described through five given points.

A. It is obvious from a figure that the system (p_1, p_2, p_3, p_4) does not, in general, contain any D.C. of the 1st kind; $\therefore \lambda_1 = 0$.

Also we can draw three pairs of lines through four given points; $\therefore$ the system contains three D.C.'s of the 2nd kind.

$\therefore \pi_1 = 3a$, where a is a positive integer.

$\therefore$ the formulae (1) and (2) give

$$\mu_1 = \tfrac{1}{3}(2\lambda_1 + \pi_1) = a,$$

$$\nu_1 = \tfrac{1}{3}(2\pi_1 + \lambda_1) = 2a.$$

Now we know that $\mu_1 = 1$, since μ_1 is the number of conics which pass through five given points.

$$\therefore a = 1, \nu_1 = 2,$$

and

$$(p_1, p_2, p_3, p_4) \equiv (1, 2).$$

B. The system (p_1, p_2, p_3, l) evidently does not contain any D.C. of the 1st kind; $\therefore \lambda_2 = 0$.

If the line p_1p_2 meets the line l in O, the two lines p_1p_2 and Op_2 will form a D.C. of the 2nd kind satisfying the conditions of the system, and there will be two others which can be obtained in a similar manner. $\therefore \pi_2 = 3b$, where b is a + ve integer.

Substituting the values of λ_2 and π_2 in formulae (1) and (2), we have

$$\mu_2 = b, \quad \nu_2 = 2b.$$

Now μ_2 is the number of conics which pass through four given points and touch a given line, *i.e.* is equal to $\nu_1 = 2$;

$$\therefore b = 2, \quad \nu_2 = 4.$$

$$\therefore (p_1, p_2, p_3, l) \equiv (2, 4).$$

C. In the system (p_1, p_2, l_1, l_2) there is one D.C. of the 1st kind, viz. the st. line which joins the two given points and is terminated at the points where it meets the given lines, and one D.C. of the 2nd kind consisting of the lines which join to the given points the point of intersection of the given lines.

$$\therefore \lambda_3 = 1 \cdot c, \quad \pi_3 = 1 \cdot d,$$

c and d being the coefficients which indicate how many times the D.C.'s enter respectively into λ_3 and π_3.

If we transform this system by means of the principle of duality we return to a system of the same kind.

$$\therefore \lambda_3 = \pi_3, \text{ and } \therefore c = d.$$

The formulae (1) and (2) give $\mu_3 = c = d = \nu_3$.

Now $$\mu_3 = (p_1, p_2, p_3, l_1, l_2) = \nu_2 = 4.$$

$$\therefore c = d = 4, \text{ and } (p_1, p_2, l_1, l_2) \equiv (4, 4).$$

D. To the system (p, l_1, l_2, l_3) belongs the D.C. of the 1st kind situated on the line joining the point p to the point of intersection of the lines l_1 and l_2, and terminated at the latter point and at the point where it meets the line l_3.

There will also be in the system two other analogous D.C.'s of the 1st kind, but no D.C. of the 2nd kind.

$$\therefore \lambda_4=3e, \quad \pi_4=0;$$

$\therefore$ by the formulae (1) and (2), $\mu_4=2e$, $\nu_4=e$.

Now $$\mu_4=(p_1, p_2, l_1, l_2, l_3)=\nu_3=4;$$

$$\therefore e=2, \quad \nu_4=2, \quad \text{and} \quad (p, l_1, l_2, l_3)\equiv(4, 2).$$

E. To the system (l_1, l_2, l_3, l_4) belong three D.C.'s of the 1st kind, which join two of the points where the four given lines intersect each other in pairs; but this system does not contain any D.C. of the 2nd kind.

$$\therefore \lambda_5=3f, \quad \pi_5=0, \quad \therefore \mu_5=2f, \quad \nu_5=f.$$

Now $$\mu_5=(p, l_1, l_2, l_3, l_4)=\nu_4=2.$$

$$\therefore f=1, \quad \nu_5=1, \quad \text{and} \quad (l_1, l_2, l_3, l_4)\equiv(2, 1).$$

11. From the values we have found for the coefficients $a, b, \ldots f$, we obtain the following rules:

In the numbers λ we must count:

Once, every D.C. joining two points where four given lines intersect in pairs and terminated at these points.

Twice, every D.C. passing through a given point and through the point of intersection of two given lines, and terminated at the latter and at the point where it meets a third given line.

Four times, every D.C. passing through two given points and terminated at the points where it meets two given straight lines.

In the numbers π we must count:

Once, every D.C. consisting of two straight lines which join by pairs four given points.

Twice, every D.C. having a double point at the point of intersection of a given line with the line joining two given points, and consisting of this latter line and the line which joins the double point to a third given point.

Four times, every D.C. having a double point at the point of intersection of two given lines, and consisting of the lines which join this point of intersection to two given points.

12. For practice the reader, by drawing a figure and using Zeuthen's rules, should obtain the characteristics in the conic systems.

Two points A, B and two tangents drawn from a point O on AB. ($\mu=2$, $\nu=2$.)

Two tangents at two given points. ($\mu=1$, $\nu=1$.)

Two points and a tangent at a third given point. ($\mu=1$, $\nu=2$.)

Three tangents and the point of contact on one of them. ($\mu=2$, $\nu=1$.)

Two points, a straight line and its pole. ($\mu=1$, $\nu=2$.)

13. In the *Comptes rendus des séances de l'Académie des Sciences*, séance of 15th February, 1864, Chasles gave, without demonstrations, a number of propositions, from which I select the following as illustrations of the simplicity and power of the theory of characteristics:

I. In a system of conics (μ, ν) the locus of the poles of a given straight line L is a curve of order ν.

When the line L meets the locus its pole lies on L, which is then a tangent, its pole being at the point of contact. Now by def. ν conics of the system (μ, ν) touch the line L. $\therefore$ the locus is of order ν.

$\therefore$ in the case of the five elementary systems the locus is (A) a conic, (B) a quartic, (C) a quartic, (D) a conic, (E) a st. line.

COR. If the line L is at infinity, its pole is the centre, and the theorem becomes:

The locus of the centres of the conics is a curve of order ν, and the curves in the different cases are as above.

In (E) the diagonals of the circumscribing quadrilateral are D.C.'s of the 1st kind; therefore the line of centres passes through their mid-points.

II. In a system (μ, ν) the polars of a given point P envelop a curve of class μ.

Let us find how often the curve-envelope passes through P. When this is the case the polar of P passes through P, which is then the point of contact of its polar with a conic. And since by def. μ conics of the system pass through P, $\therefore$ etc.

III. When the conics of a system (μ, ν) all pass through the same point P, and PP' is any straight line through P, the number of conics which touch PP' at P is $\frac{\nu}{2}$.

Let n be the number of conics required. Through P draw any other straight line PQ. The number of poles of PQ which lie on PP' (other than at P) is equal to n, there being one for each of the conics touching PP' at P. Also, the locus of the poles of PQ passes through P as many times as there are conics touching PQ at P, *i.e.* n times.

$\therefore$ the order of the locus of poles of PQ is $2n$.

$\therefore$ by I., $2n = \nu$; $\therefore\ n = \frac{\nu}{2}$.

IV. When the conics of a system (μ, ν) all touch the line QQ', and Q is any point on QQ', the number of the conics which pass through Q is $\frac{\mu}{2}$.

Use method analogous to that of III.

V. When the conics of the system (μ, ν) all pass through two points P, P' whilst satisfying two other conditions, the order of the locus of the pole of PP' is $\frac{\nu}{2}$.

By III. there are $\frac{\nu}{2}$ conics which touch at P any line through P other than PP'. And no conic through P and P' can touch PP'. $\therefore$ the locus is of the order $\frac{\nu}{2}$, and in (A) is a straight line, in (B) a conic, and in (C) a conic.

COR. If the points P, P' are at infinity, the directions of the asymptotes are given, and the locus of the centres is of the order $\frac{\nu}{2}$.

VI. When all the conics of a system (μ, ν) touch two given st. lines and satisfy two other conditions, the curve-envelope of the polars of the point of intersection of the two lines is of the class $\frac{\mu}{2}$.

Use method analogous to that of V.

VII. If from two points Q, Q' we draw tangents to each conic of a system (μ, ν) the points of intersection of these tangents are on a curve of order 3ν, which has two multiple points of order ν at Q and Q'.

Let QM be a tangent to one of the conics. The two tangents from Q' to this conic will meet QM in two points, and there are ν conics which touch QM. Therefore the tangents to these from Q' will cut QM in 2ν points. Also, there are ν conics which touch QQ', and $\therefore$ Q is the point of intersection of QQ' with the other ν tangents which can be drawn to them from Q, *i.e.* the locus is of order 3ν, and Q (and $\therefore$ also Q') are multiple points of order ν; $\therefore$ the loci in the five cases are of orders (A) 6, (B) 12, (C) 12, (D) 6, (E) 3.

VIII. If in VII. the conics of the system all touch the line QQ', the locus is of the order $\left(\frac{\mu}{2} + \nu\right)$, and has two multiple points of order $\frac{\mu}{2}$ at Q and Q'.

There are ν conics which touch QQ'. Let QM be a tangent to one of them. There are ν points of intersection of QM (other than Q) with the second tangents from Q' to these conics. Also by IV. there are $\frac{\mu}{2}$ conics which touch QQ' at Q; $\therefore$ the locus is of order $\left(\frac{\mu}{2}+\nu\right)$, and the loci in the four cases are of orders (B) 5, (C) 3, (D) 4, (E) 2.

Cor. If Q and Q' are the circular points at infinity, the points of intersection of the tangents from them are the foci of the conics. Therefore, by VII., the locus of the foci of the conics of a system (μ, ν) is a curve of order 3ν, which has two multiple points of order ν at the circular points at infinity.

Also, by VIII., the locus of the foci of a system (μ, ν) of parabolas is a curve of order $\left(\frac{\mu}{2}+\nu\right)$, which has two multiple points of order $\frac{\mu}{2}$ at the circular points at infinity.

IX. The locus of the points of intersection of tangents common to a given conic U and to each conic of a system (μ, ν) is a curve of order 3ν.

Consider any conic of the system. This and U have four common tangents. Let T be one of them. The other three cut T in three points. Now by def. ν conics of the system touch T. $\therefore$ 3ν points of intersection lie on T; $\therefore$ etc.

X. The chords common to a conic U and to each conic of a system (μ, ν) envelop a curve of class 3μ.

Use method analogous to that of IX.

14. The student who wishes to pursue the subject further is referred to *Les Lieux Géométriques en Mathématiques Spéciales avec application du principe de correspondance et de la théorie des caractéristiques à* 1400 *problèmes de lieux et d'enveloppes*, par T. Lemoyne, 10 fr., Librairie Vuibert, Boulevard Saint-Germain, 63, Paris V^e. The author gives in a table the values of the characteristics of 170 systems of conics and 40 systems of circles, each system satisfying four conditions. The work is divided into 23 sections dealing in turn with loci of poles of a given line, loci of centres of conics, envelopes of directrices when a focus is given, envelopes of asymptotes, envelopes of chords intercepted in the conics by two fixed lines, loci of feet of normals drawn to the conics from a given point, loci of vertices, loci relative to diameters, etc.

In Sections XX. and XXI. are given loci and envelopes relating to parabolas and circles, and Section XXII. deals with the determination of the values of the characteristics given in the table. Here the author, in his desire to be brief, has hardly given sufficient assistance to the student, and therefore in a new edition this section might probably be re-written with advantage.

The very large number of problems which can be enunciated is at once evident when we realise that each of the sections deals with over 200 cases in which the conditions are different, and the solutions of which can at once be obtained by simply writing down the values of μ and ν given in the table. Also, as Chasles remarked, "The theory of characteristics enables us to obtain general solutions, extremely simple, in questions many of which are quite out of the reach of the method of analysis." For instance, by its aid we can show that the loci of the ordinary foci of cubics passing through seven given points at a finite distance and touching a given straight line is a curve of the 176th order, and that the locus of the feet of normals drawn from a given point to cubics passing through five given points and touching three given lines is a curve of the 384th order. The present work, however, confines itself to loci and envelopes relating to conics and circles, and although it contains 1400 problems, this does not by any means exhaust the subject, for the student could, by means of the table, easily enunciate as many more, the solutions of which could be written down with equal facility.

J. J. Milne.

THE PRINCIPLE OF SIMILARITY.*

By W. J. Dobbs, M.A.

As teachers of elementary mathematics I suppose we are all familiar with the Circular 711 of the Board of Education. That circular is now out of print, and its place has been taken by Circular 851, which, I think, is not so well known. The purpose of the latter circular is to state the matter afresh, to explain it more fully, and to justify the attitude adopted. The main practical suggestion of these two circulars has been so largely adopted in present-day teaching in this country that I propose at the outset to base my remarks largely upon it.

§ 1 of Circular 851 reads thus: "The main doctrine of the Circular of 1909 was that certain propositions which had hitherto come at the beginning of deductive Geometry should be taken for granted as the direct outcome of intuition, and that the subsequent structure should be based upon them instead of on the ultimate postulates to which they could be logically reduced. In other words, instead of Euclid's or a more exhaustive set of postulates, these introductory propositions should themselves be taken as the basis of deduction."

Later on, in § 17 of the same circular, a dozen propositions of the first book of Euclid are selected as "Fundamental Beliefs or statements to be assumed." They include, among others, the three propositions 4, 8, 26 relating to Congruence of Triangles. To this extent, at least, a great many teachers have accepted the views expressed by the Board of Education.

Under the heading "The Congruence of Triangles" § 31 of the same circular reads: "The process of comparing two ready-made triangles, whether merely drawn or cut out of paper, is generally found to produce difficulty. A more effective method is to draw one triangle and consider which and how many of its elements must be measured to produce a copy; or better still, the question may be put in the form: 'How many elements must I give you that you may all draw the same triangle? Is the base enough? The base and the left-hand angle? The base and the two base angles?' and so on."

I want to suggest that at this early stage a teacher who adopts the method advocated by the Board of Education may quite easily and naturally introduce to his pupils not only the conditions of congruence of two triangles, but also, and almost simultaneously, the conditions of similarity of two triangles. I draw a triangle ABC upon the blackboard and invite the members of the class to reproduce it. What measurements shall I take? Three appropriate ones, one at least being a measurement of length. Suppose we decide to measure the side BC together with the angles B and C. I tell the boys that BC is $2\frac{1}{2}$ ft. long. "We cannot draw the triangle full size, sir." "No, of course not." "What size, sir?" "Please yourselves, but do not draw it too small." Several boys draw BC $2\frac{1}{2}$ inches long, a few 5 inches long, some $2\frac{1}{2}$ cm. long (too small). One makes BC 10 cm. long, and does not hesitate to refer to a length of 4 cm. as 1 ft. A few draw BC of no special length, but make it consist of 5 equal segments, calling each segment $\frac{1}{2}$ ft. Every boy knows intuitively that he can represent 1 ft. by any length he finds convenient; and when he has constructed the triangle he has no difficulty in interpreting the lengths of the remaining sides. He knows intuitively that "Any figure can be reproduced anywhere on any enlarged or diminished scale," that angles remain unchanged while lengths are all changed in the same ratio. This is the Principle of Similarity.

Now if it is agreed that certain propositions should be taken for granted as the direct outcome of intuition, and that such intuitions should themselves

* A paper read at a meeting of Teachers of Elementary Mathematics on the third of August, 1925, at Balliol College, Oxford.

be taken as the basis of deduction, what use, if any, should be made of an intuition so far-reaching as this? In a non-Euclidean Geometry similar figures do not exist. This fact alone makes a non-Euclidean Geometry unsuitable for school purposes. By assuming Euclid's parallel-postulate, or its equivalent Playfair's axiom, we are led to the doctrine of similar figures. Conversely, if we postulate the existence of similar figures we can deduce therefrom the existence and properties of parallels,—a procedure strongly advocated in the recent Report of the Mathematical Association on the Teaching of Geometry in Schools. I do not wish at the present time to criticise such a course, but I desire to suggest an alternative, and much more limited, use of the principle at one stage, and that an early stage, in a school course.

Let us return to our problem. Given a triangle ABC, we wish to make a copy $A_1B_1C_1$ on some enlarged or diminished scale, assuming the existence of such a copy. We will adopt l as the unit of length for the triangle ABC and denote its sides and angles by al, bl, cl, A, B, C, where a, b, c are numerical coefficients. In the copy we will suppose that l is represented by l_1. Then the sides and angles of the triangle $A_1B_1C_1$ will be al_1, bl_1, cl_1, A, B, C.

(i) We may measure the side BC and the angles B and C, so that a, B, C become known and the triangle $A_1B_1C_1$ can now be constructed. We may start by fixing the point B_1 and the direction of B_1C_1. Then, clearly, since l_1 is at our choice, we may make B_1C_1, just as long as we find convenient. In fact, we can construct any number of triangles $A_1B_1C_1$, $A_2B_1C_2$, $A_3B_1C_3$, ..., each of which is a copy of ABC to scale, l being represented in turn by l_1, l_2, l_3 ....

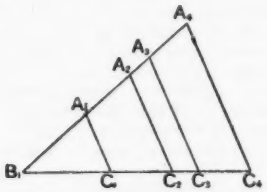


FIG. 1.

Consider any one of these triangles, say $A_1B_1C_1$. Its sides are respectively proportional to those of ABC, and its angles are respectively equal to those of ABC. It was constructed by making B_1C_1 of any length, and at the same time making two of its angles respectively equal to the corresponding two angles of ABC. Thus, clearly, sufficient conditions for the similarity of two triangles are that two angles of the one should be respectively equal to two angles of the other.

Further, if the class is already familiar with the Euclidean conditions of parallelism, it is at once seen that the lines A_1C_1, A_2C_2, A_3C_3 ... constitute any system of parallel lines.

Now B_1A_1, B_1A_2, B_1A_3 ... are respectively cl_1, cl_2, cl_3 ..., while B_1C_1, B_1C_2, B_1C_3 ... are respectively al_1, al_2, al_3 .... Thus B_1A_1, B_1A_2, B_1A_3 ... are respectively proportional to B_1C_1, B_1C_2, B_1C_3 ....

Hence the proportion properties of parallels become at once apparent.

The other two cases may be taken independently if desired, or they may be made to depend upon the case already considered, as follows:

(ii) We may construct the triangle $A_1B_1C_1$ by making two sides bl_1, cl_1 and the contained angle A, so that it would be congruent with the triangle constructed as in case (i) with the same value of l_1. But, as l_1 is at our

choice, the two sides A_1C_1 and A_1B_1 are any two lengths respectively proportional to AC and AB. Thus we conclude that sufficient conditions for the similarity of two triangles are that one angle of the one should be equal to one angle of the other and the containing sides should be respectively proportional.

(iii) Again, we may construct the triangle $A_1B_1C_1$ by making its three sides respectively al_1, bl_1, cl_1, so that it would again be congruent with the triangle constructed as in case (i) with the same value of l_1. But, as l_1 is at our choice, the three sides are any three lengths respectively proportional to the sides of the original triangle ABC. Thus we conclude that sufficient conditions for the similarity of two triangles are that the three sides of the one should be respectively proportional to the three sides of the other.

Referring to the recent *Report of the Mathematical Association* we read in § 3 of Chapter III. :

"In the course of the discussions and experiments of the last twenty years one opinion, at least, has constantly gained ground among teachers of mathematics, and may be said to be now generally received: namely, that the treatment of similar figures should hold both an earlier and a more important place than it could enjoy under the Euclidean *régime*. That opinion is accepted by the Committee as sound, and is indeed implied by their insistence upon the value, during the earlier stages, of plane-table work, model-making, and other exercises in drawing to scale. As the result of every-day geometrical experience, deepened and sharpened by such exercises, the pupil in Stage B (The Deductive Stage) should be ready to appreciate the following generalisations:

(1) Any figure (plane or solid) can be exactly reproduced anywhere. ('Principle of Congruence.').

(2) Any figure can be reproduced anywhere on any enlarged or diminished scale. ('Principle of Similarity.')

Moreover, he should be ready to formulate precisely the several sets of conditions which suffice to determine the congruence and the similarity of triangles, and to apply them in the deduction of geometrical truths."

Turning again to the Circular 851 of the Board of Education, under the heading "Proportion and Similarity" § 68 reads as follows:

"Not even the slightest course of School Geometry can be considered reasonably complete unless it includes the elements of proportion and similarity. Happily all examining bodies with one prominent exception (I think I may say here that there is now no such prominent exception) now include this section in their definition of Elementary Geometry. It is not necessary, nor even desirable, to defer it till the whole of the rest of the ground has been covered. It may be taken at any time after the establishment of the proposition that if three parallels cut off equal segments from one transversal they do from all."

I think I have pointed out a simple and natural way of taking it earlier still. Some of us are old enough to remember the time when only those boys who reached the sixth form in a Secondary school were allowed to deal with the subject of similar figures in their Geometry course, and it was considered highly improper for a boy to start any trigonometry until he was conversant with the sixth book of Euclid. The result was that the vast majority of pupils left school before they had learned to put their mathematics to practical use.

Now, definitely, my proposal is this: Let § 17 of Circular 851 include a temporary use of the intuition, "Any figure can be reproduced anywhere on any enlarged or diminished scale." I say *temporary use*, because I now go on to suggest that at a somewhat later stage some attempt should be made to

justify the intuition. Most of us, I take it, will continue to build our Geometry structure upon Playfair's axiom, and use it to prove the existence and properties of similar figures. Such a course would not involve too great a change from the customary procedure, and is alternative to the suggested course of deducing the existence and properties of parallels from the Principle of Similarity. As a boy's mathematical education proceeds he gradually ceases to be satisfied with merely testing his assumptions, and desires, if possible, and as far as possible, to prove them; and he takes an interest in doing so. How many of us older teachers attempted, while we were still schoolboys, to prove Euclid's parallel-postulate! And a boy need not go very far with his training in geometrical deduction before he is in a position to tackle the subject of the proportion properties of parallels in a manner independent of his intuition of the existence of similar figures. The child who went to the top of the mountain by rail, because the path was too long and too steep for his little legs, now, with boyish ardour, delights to make the ascent on foot, and he is the more keenly interested because it is the same mountain.

Let AB and CD be two equal segments of the same straight line, and let a system of parallels be drawn through A, B, C, D to meet another transversal

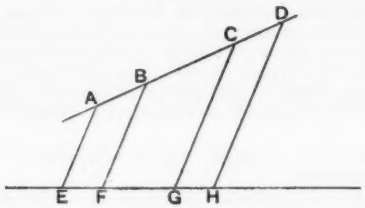


FIG. 2.

at E, F, G, H respectively. There is, of course, no difficulty in proving that $EF=GH$.

Now suppose we make successive equal segments (each c) along a straight line AB, and let a system of parallels be drawn through the points of division to meet another straight line XY. We obtain successive equal segments (each a) along XY. Suppose we divide one of the segments c into any number

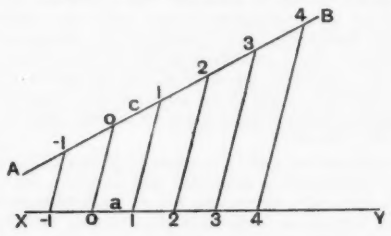


FIG. 3.

of equal parts and draw further parallels through the points of division. These additional parallels will divide the corresponding segment a into the same number of equal parts. Tenths of c along AB correspond to tenths of a along XY, hundredths of c to hundredths of a, and so on. It is at once realised that for any system of parallels, if a segment c along AB corresponds to a segment a along XY, then a segment nc along AB will correspond to a

segment na along XY, where n is any numerical coefficient, positive or negative, integral or fractional. Also a length intermediate between mc and nc along AB will correspond to a length intermediate between ma and na along XY, however near together the numerical coefficients m and n may be.

We may go a step further. Let EF and GH be any two segments of the line AB, and let a system of parallels through E, F, G, H meet XY at K, L, M, N

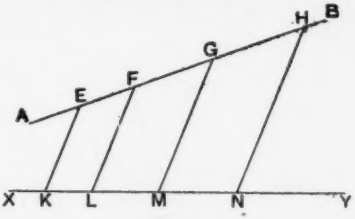


FIG. 4.

respectively. Let GH be divided into an integral number q of equal parts, and let us set off successively from E towards F parts equal to them. It may be that EF consists of an integral number p of these equal parts, but usually EF will be intermediate between p of them and $(p+1)$ of them. Now through the various points of division let further parallels be drawn to meet XY. Clearly MN is thereby divided into q equal parts, while KL, if not consisting of p of them, is intermediate between p of them and $(p+1)$ of them. Thus, if the ratio $\frac{EF}{GH}$ is exactly $\frac{p}{q}$ the ratio $\frac{KL}{MN}$ has the same value; and if the ratio $\frac{EF}{GH}$ is intermediate in value between $\frac{p}{q}$ and $\frac{p+1}{q}$ the ratio $\frac{KL}{MN}$ is also intermediate between the same two values. All this is very easy and can be made quite interesting to schoolboys. They realise that there is no approximation to the ratio $\frac{EF}{GH}$ which is not equally an approximation to the ratio $\frac{KL}{MN}$. For ordinary school purposes I do not think that we should be satisfied with anything less than this, and I do not think that we should attempt to go any further. We boldly extend our algebraical representation of numbers to include incommensurable as well as commensurable numbers, and tell our pupils that such can be added and multiplied. To a schoolboy the product of two numbers, one intermediate between a and A and the other intermediate between b and B, the capital letter in each case denoting the greater number, is a number intermediate between ab and AB, and that is sufficient. Let him think of π as a number, to which 3·14 is an approximation, 3·1416 a better approximation, 3·14159265 better still, and let him understand that as soon as he substitutes an approximate numerical value he is working approximately. Such numbers are not confined to Geometry. All surds, and nearly every logarithm and trigonometrical function he will use are incommensurable numbers, and, for school purposes, such numbers must be treated practically and not philosophically. So, in the present piece of geometry let us say that these ratios $\frac{EF}{GH}$ and $\frac{KL}{MN}$ are numbers, which cannot differ from each other in value and may be denoted by the same algebraical symbol.

Having now to this extent established the proportion properties of parallels, we are in a position to prove that any triangle can be reproduced on any

enlarged or diminished scale. For let ABC be any triangle, and along AB let any length AG be taken, drawing GH parallel to BC to meet AC, produced if necessary, at H. Also, let HK be drawn parallel to AB to meet BC, produced if necessary, at K, so that $BK=GH$. Then, by the proportion properties of parallels,

$$\frac{AB}{AG}=\frac{AC}{AH}=\frac{BC}{BK},$$

i.e.

$$\frac{AB}{AG}=\frac{AC}{AH}=\frac{BC}{GH}.$$

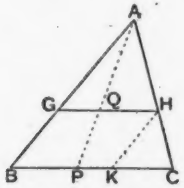


FIG. 5.

Notice, also, that if any line through A meets BC at P and GH at Q, then the ratios $\frac{AP}{AQ}, \frac{BP}{GQ}, \frac{PC}{QH}$ have each the same value as $\frac{AB}{AG}$. Also the triangle AGH is evidently equiangular to the triangle ABC. Thus the triangle ABC is reproduced as the triangle AGH on a scale in which the length AB is changed to the arbitrary length AG.

Three sets of conditions for the similarity of two triangles can now be deduced from the corresponding sets of conditions for congruence. Let ABC and DEF be the two triangles, and along AB let AG be taken equal to DE, GH being drawn, as before, parallel to BC.

(i) If it is known that $\left\{\begin{matrix}A=D\\B=E\end{matrix}\right\}$

and consequently $C=F$,

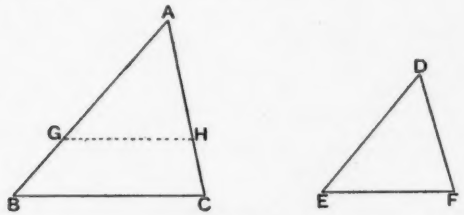


FIG. 6.

it is easily proved that $\triangle AGH \equiv \triangle DEF$.

(ii) If it is known that $\left\{\begin{matrix}A=D\\\frac{AB}{DE}=\frac{AC}{DF}\end{matrix}\right\}$,

there is again no difficulty in proving that $\triangle AGH \equiv \triangle DEF$.

(iii) If it is known that $\frac{AB}{DE}=\frac{AC}{DF}=\frac{BC}{EF}$, it follows again without difficulty that $\triangle AGH \equiv \triangle DEF$.

Thus, in every case, $\triangle ABC$, being similar to triangle AGH, must be similar to triangle DEF.

Attention should be drawn to the fact that if the sides and angles of one triangle are a, b, c, A, B, C, then the sides and angles of any similar triangle are na, nb, nc, A, B, C, where n is some numerical coefficient.

Let us consider a single example of the use of similar triangles and note its simplicity and power.

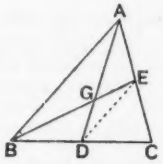


FIG. 7.

Let AD and BE be two medians of a triangle ABC meeting at G. As CD, CE are respectively the halves of CB, CA and contain the same angle, the triangle CED must be similar to the triangle CAB and of half its linear dimensions. Thus it is seen that DE is parallel to BA and equal to $\frac{1}{2}BA$. It follows that the triangle ABG must be similar to the triangle DEG and of twice its linear dimensions, so that $AG=2GD$. It appears then that the median BE cuts off one-third from the median DA. Similarly the third median CF also cuts off one-third from the median DA. Hence the three medians meet in the same point G, and the distance of G from any vertex is two-thirds of the median along which it is measured.

Now let O be a fixed point and P a moving point, and let another point P_1 be taken in OP (produced if necessary), such that $OP_1=n\,OP$, where n is a constant numerical cofficient, not necessarily positive. If the path of P is a straight line or a circle, it is interesting and not too difficult even at this stage to discuss the path of P_1, taking at first very simple cases and gradually working to harder ones. Consider, for instance, such an example as the following: Given a circle centre C and a point A outside it, show how to draw through A a straight line APQ to meet the circle at P and Q such that the chord $PQ=2AP$, and examine what limitation there is upon the position of A for the problem to be possible of solution.

It is scarcely necessary for me to point out that the treatment we have been considering is independent of areas. Such a course may indeed, if desired, precede any formal discussion of rectilinear areas or, in particular, of the area of a triangle. It is, in my opinion, better adapted for school purposes than the treatment adopted in the sixth book of Euclid. W. J. DOBBS.

---

326. Professor Heaviside was our Mathematical Pro. "Old Heavy," as he was called, was a big man, and very popular. At his lectures he used to be so engrossed in his subject that he never noticed the pranks that some of the men played, however noisy they might be; so one of them made a bet that he would drive some sheep into Heavy's lecture-room while he was lecturing, without his knowing anything about it, and he won the bet. He tipped a shepherd to let him have two or three sheep for half an hour; they were brought to the door of the lecture-room and driven in and out again, Old Heavy going on with his lecture as if nothing had happened.—*Tales Retailed*, by Sir Hastings Doyle, Bart., p. 33.

MIXTURE OF "ARABIC" WITH "ROMAN" NUMERALS DURING THE TIME OF TRANSITION.

By E. M. Langley, M.A.

Probably the most interesting example of this mixture is that given by Mr. R. Steele in the Introduction (p. xvii) to *The Earliest Arithmetics in English*, where he quotes from a British Museum MS. (Harl. 4350) a set of dates of successive births

Mij°.lviii, Mij°.lxi, Mij.63, 1264, 1266.

These form an early example, for Britain, of the transition.

Dr. J. E. Morris has drawn my attention to some curious cases of mixture in *A Copy of Papers relating to Musters, Beacons, Subsidies, etc., in the County of Northampton*, A.D. 1586-1623, now in course of publication by the Northamptonshire Record Society, and as these seem likely to be of interest to readers of the *M.G.* I shall avail myself of the courtesy of that Society to quote some of the more striking cases in illustration of the late persistence of the Roman rather than the early introduction of the Arabic.

[fo. 8] "A Letter from the Lordes of the Councell . . . To Sir Christopher Hatton, the 2° of October 1586 for the inrolinge and reducinge under captaines 1000 men . . . every band of a hundred menn to be of **xl**tie shott **xx**tie armed pickes and **xx**ltie bowes and bills the cuntrye to be charged with the trayninge of none but the 400 shott and a muster master to be chosen."

[fo. 10] "The saide chiefe traynor shall carrye the title of corporale wherof in every band consistinge or 80 or 100 persons fouer corporales shall be appointed eache to have under him **xx**tie or **xxiii** shott."

[fo. 12] "The Auncient Order of Beacons.

". . . Item for and towards the repaieracions and makinge of the seuerall beacons and the necessarye stuff therto appertaininge as also for the dayye and nightlye wages of the watchers of the saied beacons. It is ordered that everye Lord within the countye of Northampton **x**s., every Knight, **vi**s. **viij**d.; everye esquier, **5**s., every gentellman **iii**s. **iiij**d."

[fo. 17] "An: Do: 1586.

"Hundreds of Spelye **lc**lte. Wymmersley **lcl**xx**x**te. Nobottell **llciiij°**. Summa **534**."

From fo: 20 to fo. 41 in the lists of arms, men and sums of money extending over the years 1587-1597 the characters are "Arabic" except an odd **x** for 10 here and there.

There is a curious contrast between folios 45 and 46 giving rates levied on ten different hundreds for "the furnishinge and setting forth of 50 soldiours." The three hundreds on fo. 45 have the amounts stated in "Arabic," while the remaining seven on fo. 46 have all theirs in Roman, the "summa totallis of the tenn hundredes" being given as **168** li. **12**s. **2**d.

[fo. 47] in an agreement "with Captaine Parr Lane . . . for the furnishing 94: menn sent into Irelaund the 22th of September 1596.	
Allowed hym for 47 Corsletts at **40**s. a peece	**iiij**x**x** and **xiiij** li.
For three and Twentye Muskettes at **35**s. a piece	**xl** li. **v**s.
For fouer and Twentye Calyvars at **30**s. a peece -	**xxxvj** li.
For fouerscore and **xiiij**teen Coates at **xxxiij**s. **iv**d. a peece besides the Queen's allowance - -	**iiixxii** li: **xiij**s. **iiij**d.
Allowed to his soldiors **x**s. a peece - - -	**xl**tie li. **vii** li.
For a gratification to the Captaine - - -	**xxx**tie li.
For the gaole - - - - - - - -	**x** li.
Summa **319** li. **18**s. **4**d."	

A succession of folios on the supply of "corssletts and callivers" uses chiefly "Arabic" characters though a j and ij occur occasionally.

[fo. 78]-[fo. 103] stating the details of the levy for the "thirde subseedy graunted Anno 39° Eliz." have Roman numerals throughout, ending:

"Summa totalis horum decem Hundreorum **ccccix** li. **xs.**"

[fo. 107]-[fo: 137] give a similar set of accounts for the levy 1601 in Roman ending:

"Summa totall horum decem hundredorum **ccccxix** li: **viiis. viiid.**"

[fo. 184]-[fo. 189] give a statement of the levy (1612) "for the Muster Master Captaine John Fisher" in Roman with the total **72** li. **xiis.**

[fo. 303]-[fo. 309] give statements for the levy "on the Townes of the 10 Hunderds for the West Deuisione to defraye the charges of musteryng and trayning 300: foote and 50: Horsse. Perfourmed in June 1613"—almost entirely in "Arabic."

The book ends with folios 319-322, giving, in Roman, statements of assessments for taxation, 1621.

I am further indebted to Dr. Morris for the following extracts:

(i) *Lancashire Lieutenancy* published by the Chetham Society in 1859. Papers similar to those of Northants and just earlier, 1553-1594. 1588. Harl. MSS. 1926; 42, fol. 586.

P. 208, "**x**th daye of Julye until **xxx**th daye of September wch cometh to $\overline{\text{xx}}$ iiij and **ii** dayes amounteth to the some of **v** li. **ix**s. 4d.*

Manchester Division	- - - - - -	3	8 × ob. 9.
Middleton	„ - - - - - -		46 ×
Bolton	„ - - - - - -		xlvj ×

Sum **v. ix.** iiij."

* In this document the sum is miscopied from the previous entry.

Similar mixtures occur on pp. 2-8, 209, 234.

(ii) Similar Norfolk papers publ. 1907: edit. Walter Rye. 1626-1637. Pp. 149-151 from an old document of 1291 copied out in 1628:

"**ijs** . . . **vjs. 8d.** . . . . **12s. vid.** . . . . **iiis. vid.** . . . . Walsham **4s. vid.** . . . . Blofield **4s. vid.**, Lodden **5s. vid.**, Knauering **5s. vid.**"

All the numbers of villages, men, and leagues are either in "Arabic" or in "Latin" words, and of the sums of money ii and iii Roman, iiij alternates with 4 and vj with 6, while 5 and 13 are always Arabic.

There are many features of interest other than mathematical in the above *Copy of Papers*.

E. M. LANGLEY.

---

327. This most excellent Art [the Solution of Right Angled Triangles] might be much more generally understood, and used not only by Scholars, but by every common Trader, Artificer, or Husband Man, by the cheap and easy Means of a Table of *Natural Sines, Tangents* and *Secants*, and a common *Sliding Rule*.

And I am strangely surprised at the supine and stupid Indolency of many young Persons, who have (and might have by good Husbandry) Time enough on their hands, Money enough in their Pockets, and Intellects enough in their Heads, yet notwithstanding this, will expend neither in the study of this, or any other Noble Art, or Part of Mathematical Learning; which would in so agreeable a Manner, ennoble their Nature; enrich their Minds; and elevate them above, and rescue their Reputations from the rude and barbarous Vulgar. Instead of which they idly chuse to bury their Talents, and wretchedly live, and ignominously die without Remembrance.—*The Young Trigonometer's Compleat Guide, Being the Mystery and Rationale of Plain Trigonometry made clear and easy.* By Benjamin Martin (1736), p. 93.

MATHEMATICAL NOTES.

796. [L¹. 1. a.] *The Reduction of the Equation of a Central Conic to its Simplest Form.*

On page 441 of the *Mathematical Gazette* for July, 1925, in a note headed "Tracing the Conic," Mr. H. V. Mallison gives a method for tracing the central conic, which deals with an ambiguity often left undecided as to the position of the conic with regard to the axes.

The following method, which dawned upon me after nearly forty years of teaching, may perhaps be found useful by some of the readers of the *Gazette* and their pupils. There is one step in the argument which is not obvious, and consequently difficult to remember; but the ultimate advantages of taking it seem to me to be so great that I venture to send it to you, as it has, I believe, escaped the notice of the text-books.

Commencing with Mr. Mallison at the equation

$$ax^2+2hxy+by^2=1, \quad \text{.....(I)}$$

put $x=X\cos\theta-Y\sin\theta, \quad y=X\sin\theta+Y\cos\theta,$

and suppose that the equation becomes

$$AX^2+2HXY+BY^2=1. \quad \text{.....(II)}$$

Then $A=a\cos^2\theta+2h\sin\theta\cos\theta+b\sin^2\theta,$

$$2H=-2a\sin\theta\cos\theta+2h(\cos^2\theta-\sin^2\theta)+2b\sin\theta\cos\theta,$$

$$B=a\sin^2\theta-2h\sin\theta\cos\theta+b\cos^2\theta.$$

Then θ is chosen so as to make $H=0$, and *the obvious thing to do* is to express this fact by writing

$$\tan 2\theta=2h/(a-b).$$

But if it be observed that

$$A=\cos\theta(a\cos\theta+h\sin\theta)+\sin\theta(h\cos\theta+b\sin\theta)$$

and $$H=-\sin\theta(a\cos\theta+h\sin\theta)+\cos\theta(h\cos\theta+b\sin\theta),$$

then H will vanish if we write

$$\frac{a\cos\theta+h\sin\theta}{\cos\theta}=\frac{h\cos\theta+b\sin\theta}{\sin\theta}.$$

Now put each of these factors equal to λ, so that

$$a\cos\theta+h\sin\theta=\lambda\cos\theta,$$

$$h\cos\theta+b\sin\theta=\lambda\sin\theta;$$

$$\therefore\ (a-\lambda)\cos\theta+h\sin\theta=0,$$

$$h\cos\theta+(b-\lambda)\sin\theta=0.$$

$$\therefore\ \tan\theta=\frac{\lambda-a}{h}=\frac{h}{\lambda-b}. \quad \text{.....(III)}$$

$$\therefore\ (\lambda-a)(\lambda-b)=h^2,$$

$$\lambda^2-\lambda(a+b)+ab-h^2=0. \quad \text{.....(IV)}$$

When a, b, h are real, both values of λ are real.

Take *either* value of λ. *It does not matter in the least which value of* λ *is chosen*, and take the corresponding value of $\tan\theta$ from (III). This fixes the position of the X-axis without ambiguity.

The value of A is

$$\cos\theta(a\cos\theta+h\sin\theta)+\sin\theta(h\cos\theta+b\sin\theta)$$

$$=\cos\theta(\lambda\cos\theta)+\sin\theta(\lambda\sin\theta)$$

$$=\lambda.$$

The value of B is $\quad a+b-A=a+b-\lambda,$

which is the other root of (IV).

The transformed equation is

$$\lambda X^2+(a+b-\lambda)Y^2=1,$$

the position of the axis of X being fixed without ambiguity by the equation $\tan\theta=(\lambda-a)/h$, so that the conic can be traced.

Up to this point there has been no discrimination between the different kinds of central conics; but if λ, $a+b-\lambda$ are both positive, the conic is an ellipse; if λ, $a+b-\lambda$ are both negative, and this may happen when a, b, h are all real, the conic is wholly imaginary; and if λ, $a+b-\lambda$ have opposite signs, the conic is a hyperbola.

If the other root of (IV) had been taken, let it be called μ, and let ϕ be the corresponding value of θ.

Then $\quad \tan\phi=(\mu-a)/h;$

$$\therefore\ \tan\theta\tan\phi=\frac{(\lambda-a)(\mu-a)}{h^2}=\frac{\lambda\mu-a(\lambda+\mu)+a^2}{h^2}$$

$$=\frac{ab-h^2-a(a+b)+a^2}{h^2}=-1.$$

So that $\phi=\frac{\pi}{2}+\theta$.

And the new axis of X, say X', is at right angles to X, *i.e.* X' is in the same position as Y.

If Y' represent the new axis of Y, the equation is

$$\mu X'^2+(a+b-\mu)Y'^2=1,$$

which is the same as $\quad (a+b-\lambda)Y^2+\lambda X^2=1,$

as before.

39 West Heath Drive, Golders Green, N.W. 11. M. J. M. Hill.

797. [K¹. 13. c.] *The area of a triangle and the volume of a tetrahedron, obtained in Cartesian coordinates from a discussion of form.*

The Triangle. Let (x_1y_1), (x_2y_2), (x_3y_3) be the coordinates of the vertices. The area is zero if, and only if, the vertices are collinear. The required expression must contain as a factor

$$K\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}^n,$$

K and n being constants and n positive.

Let Δ be the area and P represent any other factors, necessarily symmetrical in the coordinates, so that

$$\left(\frac{\Delta}{KP}\right)^{\frac{1}{n}}=\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}. \quad \text{.....(I)}$$

If now we keep the area and two of the vertices constant, the locus of the third vertex is a straight line parallel to the base. Equation (I) must be of the form

$$a\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}+b=0,$$

and P is therefore unity. Further, since an area is quadratic in length, $n=1$. To determine K, take vertices (0, 0), (1, 0), (0, 1); the area is $\frac{1}{2}\sin\omega$;

$$\therefore\ \tfrac{1}{2}\sin\omega = K\begin{vmatrix}0&0&1\\1&0&1\\0&1&1\end{vmatrix} = K,$$

taking the vertices in the order of a positive rotation.

∴ the area of a triangle is given by

$$\Delta = \tfrac{1}{2}\sin\omega\begin{vmatrix}x_1&y_1&1\\x_2&y_2&1\\x_3&y_3&1\end{vmatrix}, \qquad \text{.....(II)}$$

which is positive if the vertices are taken in the order of a positive rotation.

Now to obtain Dr. Routh's form, given the equations of the sides,

$$a_r x + b_r y + c_r = 0 \quad (r=1,\ 2,\ 3).$$

All peculiarities are included in the cases of the lines concurrent (zero area) or of a pair parallel (infinite area), so the formula must contain the expression

$$K\begin{vmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{vmatrix}^n \Bigg/ \begin{vmatrix}a_1&b_1\\a_2&b_2\end{vmatrix}^m \times \begin{vmatrix}a_3&b_3\\a_3&b_3\end{vmatrix}^m \times \begin{vmatrix}a_3&b_3\\a_1&b_1\end{vmatrix}^m. \qquad \text{.....(III)}$$

Further, since the area might be obtained by solution for the coordinates of the vertices and substitution in (II), there will be no extraneous, always positive, factors, and (III) will represent the area for suitable values of K, n, m.

Considering the degree of the equation, we can regard at will the a's and b's as numbers and the c's as lengths; or the a's and b's as reciprocals of lengths and the c's as numbers.

The first view gives $n=2$,

and the second $6m-2n=2$;

$$\therefore\ n=2,\quad m=1.$$

To determine K, take the triangle $x+y-1=0$; $x=0$; $y=0$; of area $\frac{1}{2}\sin\omega$;

$$\therefore\ \tfrac{1}{2}\sin\omega = K\begin{vmatrix}1&1&-1\\1&0&0\\0&1&0\end{vmatrix}^2 \Bigg/ \begin{vmatrix}1&1\\1&0\end{vmatrix} \times \begin{vmatrix}1&0\\0&1\end{vmatrix} \times \begin{vmatrix}0&1\\1&1\end{vmatrix} = K,$$

and in the usual notation the area is

$$\tfrac{1}{2}\sin\omega\,.\,\Delta^2/D_1D_2D_3. \qquad \text{.....(IV)}$$

The equation gives a positive value for the area, if, as in the elementary case to find K, the sides are taken in such an order that the opposite vertices are in the order of positive rotation.

THE TETRAHEDRON. Replacing *collinear* by *coplanar*, and *straight line* by *plane*, the volume of a tetrahedron can be obtained in exactly the same way in the form

$$K\begin{vmatrix}x_1&y_1&z_1&1\\x_2&y_2&z_2&1\\x_3&y_3&z_3&1\\x_4&y_4&z_4&1\end{vmatrix}.$$

To determine K, take vertices (1, 0, 0), (0, 1, 0), (0, 0, 1), (0, 0, 0), a tetrahedron of volume

$$\tfrac{1}{6}\begin{vmatrix}1&\cos\nu&\cos\mu\\\cos\nu&1&\cos\lambda\\\cos\mu&\cos\lambda&1\end{vmatrix}^{\frac{1}{2}} = \tfrac{1}{6}\delta, \text{ say,}$$

where λ, μ, ν are the angles between the axes.

Then $$\tfrac{1}{6}\delta = K\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = K.$$

The volume of any tetrahedron is

$$\tfrac{1}{6}\delta\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}; \quad \text{.....(V)}$$

and from the order in which the vertices were taken in the particular tetrahedron, the expression is positive, provided that the order of the vertices is that of positive rotations over the faces (1, 2, 3), (1, 3, 4), (1, 2, 4), (2, 3, 4).

If the equations of the faces are given, $a_r x + b_r y + c_r z + d_r = 0$, the formula is, as before, of the form

$$K\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix}^n \Big/ \Pi\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^m,$$

for zero volume is obtained if the planes are concurrent, and an infinite volume if any vertex is at an infinite distance. Taking the a's, b's and c's as numbers and the d's as lengths, $n = 3$; and the d's as numbers and the others as reciprocals $12m - 3n = 3$, $m = 1$. To find K, take the tetrahedron given by $x = 0$; $y = 0$; $z = 0$; $x + y + z - 1 = 0$; of volume $\tfrac{1}{6}\delta$.

$$\tfrac{1}{6}\delta = K\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & -1 \end{vmatrix}^3 \Big/ \left|\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}\right| = -K,$$

and $$v = -\tfrac{1}{6}\delta\Delta^3/D_1D_2D_3D_4. \quad \text{.....(VI)}$$

Equation (VI) gives a positive value for the volume, if the faces be taken in such an order that the opposite vertices are in the order of positive rotation.

H. G. GREEN.

798. [L¹. 7. a.] *The Foci of the General Conic.*

Readers of Cremona's *Projective Geometry* know that the definition of a focus of a conic as a point at which conjugate lines are perpendicular leads to the elementary properties of the focus as directly as a definition which introduces the circular points. The same definition is equally effective in analytical geometry.

With the usual notation, two lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$ are conjugate for the conic $ax^2 + \ldots = 0$ if $Al_1l_2 + \ldots = 0$. If the lines both contain the point S, then $n_1 = -(l_1x_s + m_1y_s)$, $n_2 = -(l_2x_s + m_2y_s)$, and the condition of conjugacy is

$$Al_1l_2 + H(l_1m_2 + m_1l_2) + Bm_1m_2 - (Gl_1 + Fm_1)(l_2x_s + m_2y_s) - (Gl_2 + Fm_2)(l_1x_s + m_1y_s) + C(l_1x_s + m_1y_s)(l_2x_s + m_2y_s) = 0.$$

For S to be a focus, this condition must be identical with the condition of perpendicularity, $l_1l_2 + m_1m_2 = 0$, and therefore S lies on the two curves

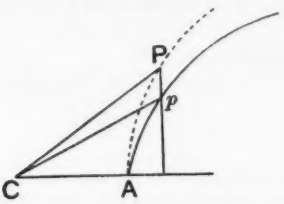
$$C(x^2 - y^2) - 2(Gx - Fy) + A - B = 0, \qquad Cxy - (Fx + Gy) + H = 0.$$

The only difference between this analysis and that which involves the circular points is that we use the symmetrical bilinear relations $Al_1l_2 + \ldots = 0$, $l_1l_2 + m_1m_2 = 0$, which are always immediately intelligible, instead of the equations $Al^2 + \ldots = 0$, $l^2 + m^2 = 0$, of which the interpretation is conventional.

E. H. N.

799. [V. 1. a.] *Speculation concerning an Imaginary Angle.*

AP is an imaginary branch of the circle $X^2+Y^2=a^2$, $X>a$.



Let us assume that

$$\text{area of circular sector } CAP=\tfrac{1}{2}a^2\phi, \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where ϕ is the imaginary angle given by

$$\cos\phi=X/a, \quad X>a.$$

Let p be the real point (x, y).

$$x=X,$$

$$y=Yi;$$

so that $$x^2-y^2=X^2+Y^2=a^2.$$

Then, since CAp is an orthogonal projection of CAP,

$$\text{area of sector } CAp=\tfrac{1}{2}a^2\phi i.$$

But $$\cosh\phi i=\cos\phi=x/a;$$

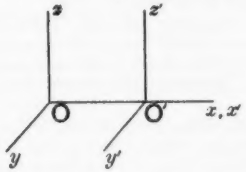
$$\therefore \text{ area } CAp=\tfrac{1}{2}a^2\cosh^{-1}x/a.$$

The truth of the result seems to justify the assumption (1).

R. W. GENESE.

800. [T. 1. a.] *Simple Derivation of the Lorentz Transformation.*

The classical theory of relativity is embodied in the equations $x'=x-ut$, $y'=y$, $z'=z$, $t'=t$. These are linear and reversible, for $x=x'+ut=x'+ut'$, etc.



Let us explore the full consequences of the hypotheses that the relations between the variables should be linear and reversible.

The first assumption is the obvious one to consider because of its simplicity; and the second, implying interchangeability of dashed and undashed variables and sign of u changed, is to be expected, because the two systems S and S' are on exactly equal footing, save that if the velocity of S' relative to S is u that of S relative to S' is $-u$.

Let, then, $$x'=mx+ut, \quad t'=px+qt.$$

The displacement of the origin O' relative to O is ut, *i.e.* when $x'=0$, $x=ut$.

Hence $$x'=m(x-ut). \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For this formula to be reversible we must have

$$x = m(x' + ut'),$$

so that $$x = m^2(x - ut) + mu(px + qt).$$

Equating coefficients, we have $q = m$ and $map + (m^2 - 1) = 0$.

The equation for t' becomes

$$t' = m\left[t - \frac{m^2 - 1}{m^2 u}x\right]. \quad \text{.....(2)}$$

This is also reversible, for, from (2) and (1),

$$m\left[t' + \frac{m^2 - 1}{m^2 u}x'\right] = m^2 t - \frac{m^2 - 1}{u}x + \frac{m^2 - 1}{u}(x - ut) = t.$$

To interpret m we note that

$$v' = \frac{dx'}{dt'} = \frac{d(x - ut)}{d\left(t - \frac{m^2 - 1}{m^2 u}x\right)} = \frac{v - u}{1 - \frac{m^2 - 1}{m^2 u}v}. \quad \text{.....(3)}$$

In classical mechanics we cannot have $v' = v$ unless $u = 0$, but here we can. Putting $v' = v \equiv c$ in (3), we have

$$c\left(1 - \frac{m^2 - 1}{m^2} \cdot \frac{c}{u}\right) = c - u,$$

so that $$(m^2 - 1)c^2 = m^2 u^2 \quad \text{or} \quad m = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}.$$

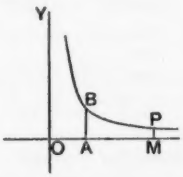
Substituting in (1) and (2) we get the usual form.

The existence of c is thus a consequence of the hypotheses "linear and reversible," and Nature (if Michelson and Morley are right) affords verification in the phenomenon of the constant velocity of light. N. M. GIBBINS.

801. [V. 1. a. λ.] The following method of treating $\frac{d \log x}{dx}$ has been used by the writer for some years. The principle is not original, being based on the treatment of the logarithmic function in Hardy's *Pure Mathematics*, but the method has been modified to make it suitable for school use. It enables the derivative of $\log x$ and e^x to be taken before the study of infinite series, which involves convergency and divergency.

The rules $dx^n/dx = nx^{n-1}$ and $\int x^n dx = x^{n+1}/(n+1)$, $n \neq -1$, have been established, also that if A represents the area under a curve $dA/dx = y$. In fact, it is introduced by considering the exceptional case of $\int x^n dx$ when $n = -1$.

To establish $d \log x/dx = 1/x$.



Plot the graph $y = k/x$. Let $OA = 1, OM = x$. Then area $ABPM$ is a function of x; denote it by $f(x)$.

Obviously $$f(1) = 0.$$

Also $$df(x)/dx = y = k/x.$$

Let u and v be functions of x.

$$df(uv)/dx=(k/uv)(v\,du/dx+u\,dv/dx)$$
$$=(k/u)(du/dx)+(k/v)(dv/dx)$$
$$=d\,.\,f(u)/dx+d\,.\,f(v)/dx\,;$$
$$\therefore\ f(uv)=f(u)+f(v)+C \text{ (some constant).}$$

When $u=1$ $\quad f(v)=0+f(v)+C\,;$

$$\therefore\ C=0\,;$$
$$\therefore\ f(uv)=f(u)+f(v).$$

This is the logarithmic law;

$$\therefore\ f(x)=\log_b x$$

(where the base b is at present unknown).

From algebra, $\quad \log_a x=\log_b x/\log_b a\,;$

$$\therefore\ d(\log_a x)/dx=k/(x\log_b a)=k'/x\,;$$

$\therefore$ the base depends upon the value of k.

Let e be the base when $k=1$, so that $d(\log_e x)/dx=1/x$.

To find e.

$$\log_e x=\log_{10} x/\log_{10} e\,;$$
$$\therefore\ 1/x=(1/\log_{10} e)\,.\,d(\log_{10} x)/dx.$$

From log. tables, $\quad d(\log_{10} x)/dx=\cdot 43/x$ approx.

$$=\cdot 4343/x \text{ approx. (using seven-figure tables).}$$
$$\therefore\ \log_{10} e=\cdot 4343\,;$$
$$\therefore\ e=2\cdot 718 \text{ approx.}$$

$y=e^x$ is then differentiated as an inverse function.

$$y=e^x\,;\ \therefore\ x=\log_e y\,;$$
$$\therefore\ 1=y^{-1}\,.\,dy/dx\,;$$
$$\therefore\ dy/dx=y=e^x.$$

The function e^x can be shown to be $\mathrm{L}_{n\to\infty}(1+x/n)^n$, *i.e.* the compound interest or growth function, only assuming that the limit of a log. equals the log. of the limit.

$$d(\log z)/dz=\mathrm{L}_{h\to 0}[\log(z+h)-\log z]/h=1/z\,;$$
$$\therefore\ \mathrm{L}_{h\to 0}\{\log(1+h/z)\}/h=1/z\,;$$
$$\therefore\ \mathrm{L}_{h\to 0}\log(1+h/z)^{1/h}=1/z.$$

Put $\quad x=1/z,\quad n=1/h\,;$

$$\therefore\ \mathrm{L}_{n\to\infty}\log(1+x/n)^n=x\,;$$
$$\therefore\ \log\{\mathrm{L}_{n\to\infty}(1+x/n)^n\}=x\,;$$
$$\therefore\ \mathrm{L}_{n\to\infty}(1+x/n)^n=e^x.$$

R. Ennis Newton.

802. [R. 2. b.] *Acknowledgement of Priority.*

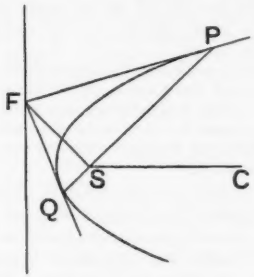
The elementary method of investigating the centroid of a uniform circular arc which I gave in the *Math. Gazette*, vol. 8 (1916), pp. 266-267 (Note 470), was published by John Dougall in *Proc. Edinburgh Math. Soc.*, vol. 16 (1898), pp. 49-50.

Wellington, N.Z. D. M. Y. Sommerville.

803. [L¹. 7. d; 3. a.] *To find the Equation of the Directrices of* $S=0$.

The polar of a point $F(x, y)$ on the directrix is perpendicular to the join of F to the focus $S(x_1y_1)$ and also passes through S.

The gradient of the polar of (x, y) is $-\frac{X}{Y}$, and the gradient of SF is $\frac{y-y_1}{x-x_1}$.



Hence
$$\frac{X}{x-x_1}=\frac{Y}{y-y_1}\equiv\lambda, \quad \text{.....(1)}$$

and also
$$x_1X+y_1Y+Z=0. \quad \text{.....(2)}$$

Again, the polar of S is perpendicular to the join of S to the centre $C(x_0, y_0)$; hence
$$\frac{X_1}{x_1-x_0}=\frac{Y_1}{y_1-y_0}\equiv\mu. \quad \text{.....(3)}$$

From (3),
$$\mu=\frac{aX_1+hY_1}{X_1-X_0}=\frac{hX_1+bY_1}{Y_1-Y_0};$$

but
$$X_0=Y_0=0;$$

$$\therefore\ \mu=\frac{aX_1+hY_1}{X_1}=\frac{hX_1+bY_1}{Y_1}. \quad \text{.....(4)}$$

And from (1), $\lambda=\dfrac{aX+hY}{X-X_1}=\dfrac{hX+bY}{Y-Y_1}$

$$=\frac{(aX+hY)(hX_1+bY_1)-(hX+bY)(aX_1+hY_1)}{(X-X_1)\mu Y_1-(Y-Y_1)\mu X_1}$$

$$=\frac{(ab-h^2)(XY_1-X_1Y)}{\mu(XY_1-X_1Y)}=\frac{ab-h^2}{\mu}, \quad \text{.....(5)}$$

since $-\frac{X}{Y}\neq-\frac{X_1}{Y_1}$, these being the gradients of PQ and the directrix, which are not in general parallel.

From (4),
$$(\mu-a)(\mu-b)=h^2.$$

Now $ab-h^2$ is the product of the roots of this equation, so that if μ is one root of this, λ is the other from (5). Hence λ also satisfies this equation.

Finally, from (1) and (2),
$$\lambda=\frac{X\,.\,X+Y\,.\,Y}{xX+yY-(x_1X+y_1Y)}=\frac{X^2+Y^2}{S}.$$

Hence the directrices are $\lambda S=X^2+Y^2$, where λ is obtained from
$$(\lambda-a)(\lambda-b)=h^2.$$

It is worth noting that equations (3) and (4) comprise a very short method for finding the equations of the axes of $S=0$. N. M. GIBBINS.

804. [J. 2. c.] *A Probability Problem.*

If two people deal out the cards face-upwards from two packs simultaneously, what is the betting against the same card falling from both packs together at least once in the course of the deal? Or, as we might put it shortly, what is the betting against a "snap"?

Calculation shows that the betting is rather more than 5 to 3 on a "snap," but it is most surprising to find that if we use packs of 51 cards by removing say the Ace of Spades, or packs of 53 cards by adding the Joker, in either case a snap becomes more likely.

If we take two packs of n cards we can suppose one pack arranged in any order, and then we must find the number of arrangements of the other pack which will avoid a snap. This is clearly the same as the number of ways in which n articles $abc \ldots z$ can be placed in n boxes $ABC \ldots Z$ without any article being placed in a box labelled with the same letter.

Let S_n stand for this number.

We can place the first article a in any one of $\overline{n-1}$ boxes, say in C. We can then place the article c either (i) in A, and then the $\overline{n-2}$ articles $bde \ldots z$ in the $\overline{n-2}$ boxes $BDE \ldots Z$ in S_{n-2} ways, or (ii) not in A, and if c is not to be placed in A the number of suitable arrangements is exactly the same as those of the $n-1$ articles $abde \ldots z$ in the $\overline{n-1}$ boxes $ABDE \ldots Z$, *i.e.* S_{n-1}.

Hence we have the formula $S_n = (n-1)(S_{n-2} + S_{n-1})$. .....(i)

This may be written $S_n - n S_{n-1} = -(S_{n-1} - \overline{n-1}\, S_{n-2})$,

so
$$S_{n-1} - \overline{n-1}\, S_{n-2} = -(S_{n-2} - \overline{n-2}\, S_{n-3})$$
$$\cdots\cdots\cdots \qquad \cdots\cdots\cdots$$
$$S_3 - 3 S_2 = -S_2.$$

Multiplying, we get $S_n - n S_{n-1} = (-1)^n$, ...(ii)

for S_2 is obviously 1.

Again (ii) may be written
$$\frac{S_n}{n!} - \frac{S_{n-1}}{(n-1)!} = (-)^n \frac{1}{n!},$$

so
$$\frac{S_{n-1}}{(n-1)!} - \frac{S_{n-2}}{(n-2)!} = (-)^{n-1} \frac{1}{(n-1)!}$$
$$\cdots\cdots\cdots \qquad \cdots\cdots\cdots$$
$$\frac{S_2}{2!} = \frac{1}{2!}.$$

Adding, we get $\frac{S_n}{n!} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots (-)^n \frac{1}{n!}$. .....(iii)

Now $\frac{S_n}{n!}$ is the chance of avoiding a snap, and is seen from (iii) to oscillate about e^{-1}, which is its limit as the number of cards in the pack increases indefinitely.

The chance of a snap with different packs is given below (correct to 5 figures).

1 card pack	1
2 ,, ,,	·5
3 ,, ,,	·66667
4 ,, ,,	·625
5 ,, ,,	·63333
6 ,, ,,	·63194
7 ,, ,,	·63214
8 (or more) card pack	·63212

It will be noticed that snap is more likely with any odd number than with any even number, and that for all numbers except 2 the betting is at least 5 to 3 in favour of a snap.

J. A. JOHNSTON.

REVIEWS.

Conformal Representation. By L. LEWENT. Translated by R. JONES and D. H. WILLIAMS. Pp. viii, 146. 7s. 6d. 1925. (Methuen.)

This is a good book, admirably translated. The author did not live to become known in England, but the familiar names of Dr. Blaschke, who added a concluding chapter to the manuscript, and of Dr. Jahnke, who edited the completed work, guarantee the soundness of the exposition.

After the necessary account of general principles, which is commendably brief, come 56 pages in which typical transformations involving only elementary functions are discussed in detail, 26 pages in which general theorems on the existence of functions for transforming a given contour into a circle and on the existence of potential functions with assigned boundary values are enunciated and explained, and 26 pages on the Schwarzian representation of convex polygons, and in particular of triangles and rectangles, on the upper half-plane. Needless to say, existence theorems are not proved; the author's merit is in so expressing himself that the most casual reader cannot imagine that such theorems are either obvious or superfluous. Possibly the allusions to connectivity are obscure without previous acquaintance with the subject, and arguments from the principles of images would be easier to follow if the language was varied more emphatically when the purpose is not to define a continuation across a boundary but only to extend the knowledge of a function upon the boundary itself. These are trifles, on which readers may not agree. The important fact is that readers will certainly become familiar with the leading ideas of a fascinating subject, and will understand them.

The step from familiarity to mastery is largely a matter of practice, and it is here that the book, being without examples and practically without references, may fail to suit the class of readers to which the publishers specifically commend it, namely, physicists, engineers, and cartographers who need conformal representation as a tool. But if there is a public in England for books on relativity which are serious without being profound, may there not be a public also for accounts of the more picturesque branches of pure mathematics? For this public, examples would be an imposition, and references should be confined to a few standard works. In fact the volume under consideration has precisely the qualities to imply that its success or failure will go far to answer the question suggested, and if the answer is favourable, perhaps some enterprising publisher will even invite English mathematicians to be authors rather than translators. E. H. N.

Notations et Formules Vectorielles. By A. LAFAY. Pp. vi, 36. 6 fr. 1925. (Gauthier-Villars.)

This synopsis was written to enable students of physics to read work in which vectors are employed. No applications are described, and the algebra does not extend beyond the rudiments—the dyad does not appear—but in the direction of analysis the definitions and formulae cover in much the usual way the ground as far as the expression of an arbitrary field inside a closed surface by means of surface and volume integrals. The author adopts Résal's notation, and although he gives in footnotes various equivalent symbols in current use—without, however, mentioning Gibbs' use of a dot to distinguish the scalar product—one cannot but feel that Prof. Lafay's aim is so far incompatible with the use of any system which is not already well established and widely used that criticism of details is beside the point. E. H. N.

Lecons de Géométrie Élémentaire. By É. A. FOUËT. Pp. xv+348. 1924. (Vuibert, Paris.)

The author is divided between a love of geometry and of Euclid, and a loyalty to the French government syllabus, which deprives his book of part of its value for foreigners. He disarms any criticism of detail by announcing that he does not intend to mention obvious exceptions: this may justify the statement that the inverse of a plane is a sphere, but not that inversion was first studied by Plücker, nor that a *faisceau* of curves is "a set enjoying

certain properties." On the whole, there is a sensible alternation of real proofs, some very neatly worded, with slighter indications; the choice of topics is reasonable and varied. M. Fouët steadily tries to make them interesting, and has had the self-control to keep the book short. Perhaps the historical notes are too short and numerous: none of the mathematicians named is made to seem a human being.

The book is charming because of a certain modesty and enthusiasm in the author that one hopes will infect all his readers. H. P. H.

Curve Sghembe Speciali, algebriche e trascendenti. By GINO LORIA. Vol. I. **Curve algebriche.** Pp. xi+374. L. 65. 1922. (Zanichelli, Bologna.)

The first fifty-eight pages of this book are devoted to a summary of the principal formulae relating to general twisted curves and an account of researches into their properties. The description is often encyclopedic in style, proofs are frequently, though not always, suppressed, and an attempt is made to treat the matter exhaustively. With Chapter III. begins the discussion of algebraic curves. After ten rather superfluous pages on the conic appears a very thorough and readable treatment of the twisted cubic with complete proofs. The cubic hyperbola is accorded a detailed discussion; with the other special cubics the author deals, less fully.

To each type of quartic curve is assigned a chapter, also very useful and interesting. The final chapter on algebraic curves of degree higher than four is too scrappy to be of much value, but doubtless little is known about such curves.

The argument is almost entirely analytical, and the book contains only some three or four diagrams. Perhaps the analysis is a little overdone; the section on sphero-conics, for instance, could have been made far more concise and interesting by a more liberal use of projective methods.

A serious blemish in a work primarily devoted to algebraic twisted curves is the fact that the definition of such curves is shirked. Thus Chapter IV. begins, "A twisted curve of the third degree is met by any plane in three points," and similarly for quartics on p. 204. The difficulties in the way of a logical definition are serious, but some attempt more courageous than the above was needed.

The book is clearly printed, and the style is very pleasant and intelligible. Misprints are not over-frequent. They occur mainly in quotations from foreign languages (*e.g.* "A Treatise on some news geometricals Methods"), and in printing ∞' for ∞^1 in nearly every case. The references are numerous and useful; there is a list of contents, but no index.

The book will be read with pleasure and profit by all mathematicians interested in this kind of geometry, and they should lose no time in purchasing a copy. HAROLD HILTON.

The Theory of Relativity. By L. SILBERSTEIN. Second edition, enlarged. Pp. x+563. 25s. net. 1924. (Macmillan.)

The first edition of Dr. Silberstein's book appeared in 1914, and consisted of a very full account of what is now called the restricted or special theory of relativity. Of course the general theory did not exist at that time. The second edition is nearly twice the size of the first. The earlier matter is reprinted, with scrupulous care to bring it up to date by means of foot-notes, and there has been added a detailed account of the general theory and of its cosmological extensions.

The book is on a large scale, and it contains a great deal that cannot be found in any other treatise. The most original part is the very long concluding chapter entitled Cosmological Speculations. As this deals with a topic that is understood by few, it seems worth while to discuss it in some detail.

In spite of the triumphs of his gravitational theory of 1916, Einstein himself was dissatisfied with it on three grounds:

(i) Although the differential equations were independent of the choice of coordinates, these equations had to be supplemented by conditions at infinity expressed in a special coordinate system.

(ii) The inertia of a particle appeared still as an absolute property of the particle, whereas Einstein, following Mach, thought that it should be determined by the combined effect of all the matter in the universe.

(iii) The universe appeared to have a tendency to "run down." Its radian, energy and even the heavenly bodies themselves would, from time to time, go off to infinity and never return.

It was found later that the first difficulty is easy to overcome, and that the conditions at infinity can be expressed in a form independent of the coordinate system. A method of dealing with the third difficulty is indicated by some work of Carl Neumann and Seeliger on Newtonian potential. However, the second difficulty remains, and this is the one to which Einstein attaches most importance.

His method of escaping from this difficulty is characteristic in its boldness. It is to abolish infinity altogether and to assume that the universe is finite. At first sight such a theory seems absurd, and contrary to what Euclid has proved. However, we need to realise that our school geometry is in fact a somewhat confused combination of two entirely different subjects; one an exact but purely abstract science dealing with mysterious entities called "straight lines" which are really undefined, except in a vague way by the axioms, which are the statement of the properties to be taken as the basis of the reasoning; the other a physical science dealing with the properties of certain pencil marks and the edges of rulers, which are what we usually mean when we speak of "straight lines." It is true that the most careful drawing and experiment have as yet failed to reveal any difference between the laws of these two sciences, but this may be due merely to the smallness of this difference and not to its non-existence. A difference that is negligible on a terrestrial scale may become of great importance on an astronomical scale, and so there is nothing inherently absurd in Einstein's assumption that the physical universe is finite and of the type discussed in non-Euclidean geometry under the somewhat misleading name of Elliptic Space. It is easy to become confused with the properties of such a space, so we merely note that the path of a ray of light or of a moving particle will, although never coming to a boundary, nevertheless return on itself, like a great circle on a sphere.

This theory satisfies all the conditions required, provided that the average density of the matter filling space is taken to be constant, and not zero, with a higher value at the comparatively few points occupied by the heavenly bodies. This suggestion that space contains vast quantities of matter not detected by astronomers is startling, although L. V. King, by considering certain selective attenuation effects on starlight, has inferred that interstellar space contains stray molecules of gas of a density much greater than the average over all stellar space of the density due to the stars themselves. But later work by Shapley throws grave doubt on King's conclusions. Dr. Silberstein declares that Einstein's assumption of a finite universe and of a non-zero density is "perfectly arbitrary," and replaceable by that of an infinite universe with re-entrant light paths, and with a zero density of matter (except at the heavenly bodies).

The real difficulty of Einstein's cosmological theory arises when the fields due to massive bodies (such as the sun) are considered. The field equations become inconsistent with one another, unless the mass is zero, or the universe is infinite. To make the equations consistent we may introduce a pressure, but, as Dr. Silberstein shows in great detail, this leads to further difficulties. Space will then contain a number of impenetrable barriers, in addition to the heavenly bodies of known type. As there is no evidence of the existence of these barriers, the theory which implies their occurrence is "completely indefensible." Most English writers have expressed (in milder language) similar objections to Einstein's cosmological theory, but continental writers regard it much more favourably.

An alternative theory is proposed by de Sitter. He gives up the Mach-Einstein postulate of the complete relativity of inertia and replaces it by a "mathematical postulate," of which, unfortunately, it is difficult to give any physical interpretation. The universe is again conceived as finite, but with no matter in it except the heavenly bodies. The field equations for a massive

body are quite consistent, giving Newtonian dynamics as a first approximation, and the usual perihelion motion, bending of light, and spectral shift as a second. The impenetrable barriers seem to occur again, but in this case they are merely a mathematical singularity of the co-ordinate system, without any physical significance. (In the original form of Einstein's cosmological theory, without massive bodies, there are no barriers, real or imaginary.) Another apparent peculiarity of de Sitter's world, the existence of places where a " death-like standstill " prevails and all events take an infinite time, is also merely a mistaken inference from a mathematical singularity of the coordinate system.

The most attractive feature of de Sitter's theory is that it makes certain predictions that can be tested by astronomical observation. In Einstein's universe a particle will remain at rest wherever it is placed, but in de Sitter's universe it will, according to Eddington and Weyl, be repelled outwards with an acceleration increasing with the distance. Owing to this tendency to scatter, and also to another feature of de Sitter's theory, the slowing down of atomic vibrations at a great distance, the spectra of very distant objects, such as the spiral nebulae, should show a displacement towards the red. This is precisely what is observed in the great majority of cases, but there are some disconcerting exceptions. However, Dr. Silberstein declares that this tendency to scatter is merely " a hasty conclusion " of Eddington's, inspired by " a conscious but perfectly gratuitous assumption " of Weyl's. The restlessness of particles is admitted, but a calculation is given that seems to show that the tendency is to gather more than to scatter. Dr. Silberstein works out his views in some detail, and brings forward astronomical evidence which seems very favourable. But it is quite possible that the eminent authorities whom Dr. Silberstein criticises so freely may find some points open to criticism in his own work.

To sum up, de Sitter's theory appears to contain the promise of interesting astronomical developments, and Dr. Silberstein's contributions are full of interest and likely to stimulate further research.

The other parts of the book will now be noticed very briefly, as they cover well-known ground, though the treatment is often so different from that of other writers that the subject appears in a new light. Chapters I.–X. are reprinted from the first edition, so the only comment needed is an appreciation of the valuable notes on recent work, including a preliminary account of the experiments of Michelson and Gale, and of Dayton C. Miller. (If Miller's results are confirmed, the original foundations of the theory of relativity seem to be shattered. The correspondence columns of *Nature* contain interesting letters on this subject from Dr. Silberstein and others, but it is evident that there is a strong conflict of opinion.) Chapter XI. is a very readable account of the fundamentals of the gravitational theory, emphasising the physical aspect and omitting mathematical complications. The remaining chapters deal respectively with tensors, geodesics, gravitation, electromagnetism, and cosmological speculations. The book concludes with a set of miscellaneous notes and an exhaustive index.

The student who has read Eddington's *Mathematical Theory of Relativity* and wishes for a supplementary treatment will welcome Dr. Silberstein's book. Its highly critical tone and its detailed discussion of aspects usually neglected are not suitable for beginners, but it is very valuable and stimulating to those who are familiar with the usual presentation and desire another point of view, not for passive acceptance, but as a stimulus to independent thought.

H. T. H. PIAGGIO.

Scientific Papers, mainly on Electrodynamics and Natural Radiation, including the Substance of an Adams Prize Essay in the University of Cambridge. By the late SAMUEL BRUCE M'LAREN. Pp. vii + 112. 8s. 6d. net. 1925. (Cambridge University Press.)

This book is not a mere reprint of the published works of Professor M'Laren ; it is really a memorial volume to one whose best work was cut short by the outbreak of war in 1914, and his death from wounds two years later. In addition to an account of his scientific work, the book also contains a short

biography by Professor Hugh Walker, who gives us a clear view of the personal side of one who is an irreparable loss to the future of Mathematical Physics in this country.

Immediately before the War, M'Laren was engaged in preparing for the press his successful Adams Prize Essay of 1913 on the Theory of Radiation. In the first section of this book there is an adequate account of this work, in so far as it has not been superseded by more recent investigations. The problem of energy partition is discussed with great analytical skill, and the conclusion arrived at is that, if the Hamiltonian Equations as derived from Least Action and an essentially positive energy function are used, the equipartition must follow. Then comes an attempt to save the classical view of radiation as a continuous wave motion by sacrificing the formulae of classical dynamics. By making certain special assumptions, but not using the Principle of Least Action, it is shown that the wave theory of radiation can lead to a formula similar to that of Planck. In the last paper of this section there occurs a highly speculative endeavour to include Gravitation in an Electromagnetic scheme, matter being regarded as an aether sink.

The second section of the book on Electromagnetic Theory is chiefly of interest because here is developed the theory of the Magneton, for which M'Laren is, perhaps, best known. Rejecting entirely the idea of magnetic or electric substance, he supposed that matter is merely the inner boundary of the region in which the Maxwell Equations hold, and showed that there is an essential difference between simply- and multiply-connected bounding surfaces. The simply-connected boundary behaves just like an electric charge; but when the inner limiting surface of the aether is multiply-connected, *e.g.* like an anchor ring, the tubes of electric force which terminate on its surface give it a charge, while the magnetic tubes through its aperture make it a permanent magnet. The latter type of boundary is the Magneton.

In the last section is published for the first time the substance of a group of papers, left incomplete by M'Laren, dealing with the propagation of an electromagnetic disturbance in a particular type of dispersive medium; a comparison is made between the mathematical analysis by Fourier integrals and the physical separation of an impulse into waves and groups of waves. Solutions of the partial differential equations involved are given in the form of complex integrals, and approximate results are obtained by the use of what is now called the method of steepest descents. It cannot, however, be conceded that this constitutes "an independent study in the method," as the Editors claim; for M'Laren must have known the work of Debye in 1909 and 1910, at least three years before.

The book has been produced by the Cambridge University Press in the good style which one always expects of them; but it is to be regretted that a little more care in proof-reading was not taken. The Editors, however, have been extremely successful in their attempt to produce a volume which should serve as a tribute to M'Laren's memory and a memorial of his scientific activity. E. T. COPSON.

Intermediate Mathematics (Analysis). By T. S. USHERWOOD and C. J. A. TRIMBLE. Pp. xii + 454 + xxiv. 7s. 6d. 1925. (Macmillan & Co., Ltd.)

This book is intended for those who are reading mathematics of the Intermediate Standard for a degree in Arts, Science or Engineering. It begins with revision of some quite elementary Algebra and Graphs, and a chapter on Indices and Logarithms. There are then 50 pages of Trigonometry, followed by 50 pages of Coordinate Geometry. Some further Algebra is then given, including an introduction to Determinants, Progressions, and the Binomial Theorem. There is also a chapter on Vectors. The Trigonometry is then extended to the end of solution of triangles, and the book closes with an introduction to the Calculus. The general style compares very favourably with that of many books intended for more purely mathematical readers. Details about Convergence would be out of place in such a book, but when Infinite Series occur, the necessary facts are clearly and correctly stated. The theory of logarithms is concisely set out on p. 95; it is curious that the authors should have reverted on p. 109 to the comparatively clumsy method

common to most text-books. If the section on Determinants had been placed earlier, more use of it could have been made in the Coordinate Geometry. The chapter on Vectors is very welcome; it reaches so near to the stage at which the (projected) product can be introduced that it is to be regretted that it did not extend a little further. The last 90 pages of the book contain a good introduction to the Calculus. The collection of examples is a good one.

Elementary Algebra. PART I. By F. BOWMAN, M.A. Pp. xii + 636. 6s. 1925. (Longmans, Green.)

The author has made it his first object to unfold the argument in a logical order, and his second object to enable the student as soon as possible to apply his knowledge to the problems of science and engineering.

Part I. extends as far as the (elementary) Binomial Theorem and, up to that point, there is little omitted which a mathematician could expect to find in an elementary text-book.

Graphs are not reached until p. 360; their treatment is more systematic and satisfactory than usual, and there are interesting chapters on quadratic functions, polynomials, and rational functions, in which graphical representation predominates. If, however, an earlier chapter on the elements of graphs had been included, illustrations of the solution of simultaneous equations would have been available to improve the otherwise admirable articles 24, 25, 75. Moreover the chapter on Variation could then have been made much more attractive.

Indices and Logarithms are also postponed to a late stage.

The treatment of quadratic equations is divided into three parts, and before the second part there is a chapter on quadratic surds and complex numbers. The treatment of complex numbers is unsatisfactory, not because it is incomplete—completeness need not be demanded at this stage—but because it will give the beginner wrong ideas on the subject.

It is unfortunate too that, in the chapter on rational functions, some vague conventions about $x=\infty$ are introduced. For example the inconsistency of the equations $x-2y=-1$, $x-2y=2$ is "removed" by the convention that their solution is $x=\infty$, $y=\infty$, and this is excused by the consideration that $mx-2y=-1$, $x-2y=2$ have the solution $x=3/(1-m)$, $y=(2m+1)/(2-2m)$ when m is not 1. If it is desirable to "remove the inconsistency" the appropriate equations to be considered are surely $x-2y=-z$, $x-2y=2z$ with their solution (2, 1, 0). One geometrical illustration that is given is more likely to be misleading than helpful; for it is not explained that the alternative definitions of parallel lines belong to different kinds of Geometry, nor is there any limit that "meet at infinity" requires definition. There is a short section on nomographs.

The Binomial Theorem is deduced, in a novel way, independently of the theory of Combinations. The book contains upwards of 4000 examples, with answers. A. R.

Machinists' and Draftsmen's Handbook. Second edition. By P. LOBBEN. Pp. x + 48, 87. 10s. 6d. net. 1925. (Harrap.)

Operating Engineer's Catechism of Steam Engineering. By H. M. GORNSTON. Pp. vi + 428. 12s. 6d. net. 1925. (Harrap.)

Mr. Lobben's volume is well printed and nicely arranged. There is a very useful collection of reference tables. The sections dealing with mensuration, strength of materials and mechanics are excellent. To us in this country U.S. standards are but of occasional interest. The Steam Table (No. 82) is carried only to 185 lbs. per sq. inch gauge pressure, and the source of information is not given. The comparison of Thermometric Scales in Table 84 is of limited application. The Reaumur Scale might well have been omitted, and the comparison of the Fahr. and Cent. Scales might have been carried to 850° F.

As the title implies, Mr. Gornston's Catechism is throughout in the form of question and answer. It covers a large field and is non-technical—in fact so non-technical in places as to be scientifically inaccurate. The author has clearly had no experience of the modern steam turbine, for this section of the book is obsolete and misleading. F. C. M.

A First Trigonometry. By W. G. Borchardt & A. D. Perrott. Pp. x + 114 + xvii + xii. 2s. 6d. 1925. (G. Bell & Sons.)

A book of limited range, suited for those who are not likely to require the use of Trigonometry beyond the solution of right-angled triangles. The ratios are introduced one by one, each new ratio bringing with it a new series of problems.

More stress might have been laid on the use of the ratios as "multipliers"; the student of elementary mathematics cannot be taught too early to write straight down: "$b = 12 . \sec 30°$," instead of working through the cumbrous argument:

$$\frac{12}{b} = \cos 30°, \qquad \therefore \ b = \frac{12}{\cos 30°}.$$

There is some looseness of language, such as, "It is easily proved experimentally"—(no young person should be allowed to think that such experiments as wrapping cotton round a cylinder *prove* anything)—"$\tan 90° = \frac{1}{0} = \infty$."

"$\log y = \cdot 2455$
$\cdot 4655$
$= \cdot 7110$."

In particular, the arrangement of logarithms in the worked-out examples is not happy; logarithms are most conveniently placed in columns in the margin, and not in lines in the text.

The last chapter deals with circular measure. In Trigonometry, where the idea of ratio is running through the whole book, it seems a pity to miss the opportunity of defining circular measure of an angle as the ratio between the arc and the radius, instead of as "the number of radians." In fact, this chapter as it stands seems to have little connexion with Trigonometry, and belongs rather to elementary Mensuration.

The examples are well chosen and varied, and the printing is excellent, save in the Graphical diagrams of Chapter X., where the graph line is so thick that a 5 per cent. error might be made in reading the graph; this is the more noticeable in that the graphs in the earlier chapters are excellently reproduced. The printing and arrangement of the tables are good, but it seems unnecessary nowadays, in view of the improvement in printing, to perpetuate the old-fashioned habit of adding 10 to the logarithm of a trigonometrical ratio.

Junior Mathematics. By F. M. Saxelby. Pp. vi + 181. 2s. 6d. 1925. (Longmans, Green & Co.)

This is a short course intended for pupils in Continuation Schools, etc., and is admirably suited for the purpose.

The first three chapters give a general revision of decimal work—an excellent treatment of "Graphic Arithmetic," and an introduction to logarithms which should be sufficient to secure the necessary ability in the handling of these aids to calculation.

Standard form is used freely in decimal work, but it is better to avoid the phrase "Move the decimal point." Even for the type of student for which this book is intended it is better to move the figures relative to a fixed decimal point.

A short section introducing algebraic method is equally satisfactory, and the elements of mechanics and trigonometry are dealt with in conclusion.

Principles of Arithmetic. By P. J. Harwood. Pp. 112. n.p. 1925. (Methuen & Co., Ltd.)

This is a book with an attractive title which is not justified by the contents. The treatment offers little that is new, and there is much that is distinctly retrograde. It is, for example, essential in modern arithmetic that decimals should be placed on the same footing as whole numbers, so that the method employed in the manipulation of whole numbers may be used with slight modifications in dealing with decimals; and yet it is suggested that "the meaning of" (the multiplication of two decimals) "can best be understood by changing to and working in vulgar fractions."

The treatment of vulgar fractions themselves is unconvincing. In a work which purports to establish principles, one would naturally look for the main principle, "a fraction is unaltered if its numerator and denominator are each multiplied or divided by any one quantity"; instead of this simple principle we find the complicated "rule": "To change a fraction from one denomination to another denomination the denominator of which is a multiple of that of the former, divide the old denominator into the new denominator and multiply the result by the old numerator to produce the new numerator."

There is also an unsatisfactory discussion on "cancelling." The language is often unnecessarily complex and sometimes inaccurate; *e.g.* "compound quantities are constructed on compound, irregular and discontinuous scales, so that their parts are so many heterogeneous jointings." "While addition means making *more*, subtraction means making *less*."

"An average may be defined as the equal representative number of unequal numbers."

There are a few exercises at the end which do not add to the value of the book.

Algebraic Geometry. By M. P. MESHENBERG. Pp. xi + 127. 3s. 6d. net. (Sidgwick & Jackson.)

The author has succeeded in compressing into a small space a very large amount of the theory of Algebraic Geometry. It is unusual in an introductory work such as this to find a fairly full treatment of co-axal circles, poles and polars, parametric and abridged notations, and the inclusion of these subjects adds considerably to the value of the book. Valuable hints are given on the solution of many of the examples, but the number of exercises is not sufficient to give the reader any great facility in manipulation, and the book cannot in consequence serve as more than an introduction to the subject.

The author has deliberately avoided the use of the Calculus, and, although many teachers will not agree with him in thus discarding the most powerful weapon of mathematics, he has undoubtedly presented his subject in an attractive form.

No index is given—again deliberately—but each chapter is provided with a full table of contents and an excellent summary.

There is too great a tendency to use phrases such as "It is easily shown," "It is obvious that," etc. Students should not be encouraged at this stage to think that anything is obvious. Great care has been taken by the use of varied type to bring out important points, but the variation has been overdone and some of the heavier type used is unsuitable. Most of the examples are in small type that is trying to the eyes and is often defective, notably on page 26, line 22.

The whole book shows a thoroughness of treatment which should commend it to those who require as full a knowledge of the subject as can be obtained in a limited time. N. J. C.

(1) **Summation of Series.** Compiled by L. B. W. JOLLEY. Pp. x + 232. 13s. 6d. net. 1925. (Chapman & Hall.)

(2) **Test Papers in Arithmetic.** With Answers. For the Use of Candidates preparing for School Certificate, Matriculation and similar Examinations. By A. E. DONKIN. Pp. iv + 119. 2s. 6d. 1925. (Pitman.)

(3) **Miscellaneous Exercises in School Mathematics.** Compiled and arranged by H. E. PIGGOTT and D. F. FERGUSON. Pp. vii + 250 + xxxii. 4s. 6d. Part I., 2s. 6d. 1925. (Bell.)

(4) **Arithmetic.** Parts I. and II. By C. V. DURELL and R. C. FAWDRY. Pp. xiv + 156 + 20. 2s. 1925. (Bell.)

(5) **Building Mathematics.** Part I., Part II. By R. W. M. GIBBS. 1s. 3d. each. 1925. (Blackie.) And in one vol., Parts I.-III. Pp. 192 + xi, 4s. 1925. (Blackie.)

Engineering Mathematics. Part II. By R. W. M. GIBBS. Pp. iv + 65 to 128 + viii. 1925. (Blackie.)

(1) Mr. Jolley has found a demand for a collection of series for easy reference, and the 700 in this volume have been selected from twenty text-books ranging

from elementary works to *Whittaker and Watson* and the *Smithsonian Formulae*. Reference is given in each case to a text-book which will indicate the method by which the series and its sum were arrived at. The insertion of blank pages affords those engaged in the solution of technical problems an opportunity of inserting in their appropriate place such series as they come upon in their special line of investigation and consider worthy of preservation. The last eleven sections are headed respectively: Taylor's and Maclaurin's Theorem; Bessel Functions; Elliptic Functions; Various Integrals; Gamma Functions; Infinite Products; Fourier's Series; Relations between Products and Series; Special Functions; General Forms; Bernoulli and Euler Numbers. The series are well-spaced and clearly printed.

(2) Long experience as a master in a Public School and as Examiner for the O. and C. Joint Board, London, and the Northern Universities, has enabled Mr. A. E. Donkin to construct just the right type of test paper for students preparing for School Certificate, Matriculation and similar examinations in which "a definite standard of arithmetical knowledge is demanded." The papers are each of about the same difficulty, and the candidate who does a couple a week in the year ending with a Matriculation should have no qualms in facing any question with which the ingenuity of examiners may confront him. The last couple of dozen papers are especially constructed to include questions on logarithms and easy numerical trigonometry to "meet the requirements of any school which includes these subjects in its syllabus of arithmetic." The questions are remarkably varied, and for the most part original; answers are supplied to all. The papers are clearly printed, one to each page.

(3) Messrs. Piggott and Ferguson have divided their sets of revision papers into four Stages. Stage A (Arithmetic, Algebra, and Geometry) are elementary. Elementary Trigonometry is added to the subjects in Stage B, and the standard is about that of the School Certificate and "Elementary Mathematics" of the Army Examination. Stage C covers the ground of the "Intermediate Mathematics" for the Army; and C and D together supply the needs of candidates taking the higher papers in the School Certificates, "More Advanced Mathematics" for the London Matriculation, and the "Higher Paper" for the Army. The examples are grouped under subjects. A supplement is added for the benefit of the best pupils, suggesting various "investigations" within their reach. With certain stated exceptions the questions are original, and many, especially among those on the Calculus in Stage D, are of considerable ingenuity and interest.

(4) This little volume contains two of the three parts of a text-book intended for those who have learned the earlier part of the subject, and need instruction in "the best methods to adopt in simple computation." Revision papers supply practice in rapid computation, and Problem Papers provide practice in grasping the meaning of questions. An interesting feature is the judicious use of historical notes. None of the hints so necessary at this stage of the pupil's progress seems to be forgotten.

(5) The rest of the books in the above list do not seem to require further comment than that they are carefully thought out, and should meet the requirements of the budding builders and engineers for whom they are intended.

---

328. The Countess of Dysart ... had a wonderful quickness of apprehension ... had studied not only divinity and history, but mathematics and philosophy.—Bishop Burnet's *History of His Own Times.*

329. The Earl of Essex ... made himself master of the Latin tongue, and attained a considerable knowledge in mathematics.—Bishop Burnet's *History of His Own Times.*

330. The chief of our plenipotentiaries [at Ryswick] was the Earl of Pembroke, a man ... of great and profound learning, particularly in the mathematics. This made him a little too speculative and abstracted in his notions. —Bishop Burnet's *History of His Own Times.*

THE MATHEMATICAL ASSOCIATION.

VICTORIA BRANCH.

We have had a fairly successful beginning, and have had meetings as follows:

(1) The President (Professor J. H. Michell), "Mathematical Scandals" (*i.e.* Stumbling Blocks).

(2) Discussion on Elementary Teaching of Calculus, introduced by Mr. D. K. Picken (two evenings).

(3) Notes on Greek Mathematics (especially on Arithmetic, by Mr. R. J. A. Barnard.

(4) Discussion of Report on Geometrical Teaching, introduced by Miss J. Flynn (two evenings).

A. J. Barnard, *Hon. Sec.*

THE PILLORY.

"... deduce that the three medians of a triangle are concurrent and are trisected at the point of concurrence."—London Matric. Geom. Paper, June 1925.

"To trisect," according to the dictionaries, is "To cut or divide into three equal parts."

How can a straight line be trisected at one point?

[The phrase is not uncommon, although of doubtful propriety.

I find in Mackay's *Euclid*: "The centroid may be found by drawing one median and trisecting it."

In Godfrey and Siddons' *Modern Geometry*: "Each median is trisected at the point of concurrence." "Prove that the medians trisect one another."

W. J. G.]

THE TEACHING OF GEOMETRY IN SCHOOLS.

The supply of this Report having been exhausted, opportunity has been taken to make a few slight changes. A list of these amendments, with a copy of the prefatory note to the Second Edition, is issued with this number of the *Gazette* for the sake of readers who have the Report in its original form.

THE LIBRARY.

160 Castle Hill, Reading.

Donations of back numbers of the "Gazette" are always welcome.

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD.
AT THE UNIVERSITY PRESS, GLASGOW

BOOKS RECEIVED, CONTENTS OF JOURNALS, ETC.

October, 1925.

Aufgaben und Lehrsätze aus der Analysis, II. By G. POLYA and G. SZEGO. 10-407. 15 Goldmark. 1925. (Springer, Berlin.)

Notations et Formules Vectorielles. By A. LAFAY. Pp. 33. 6 fr. 1925. (Gauthier-Villars.)

Principles of Arithmetic. By P. J. HARWOOD. Pp. 112. n.p. 1925. (Methuen.)

Junior Mathematics. By F. M. SAXELBY. Pp. vii + 182. 2s. 6d. 1925. (Longmans.)

Mathematical Association. Yorkshire Branch. Session 1924-1925. Pp. 16. (Jowett, Leeds.)

The Number System of Arithmetic and Algebra. By D. K. PICKEN. Pp. viii + 76. 5s. 1925. (Macmillan.)

Historic Instruments for the Advancement of Science. A Handbook to the Oxford Collections, prepared for the opening of the Lewis Evans Collection on May 5, 1925.) By R. T. GUNTHER. Pp. 90. 2s. 6d. net. 1925. (Oxford University Press.)

Précis de Mécanique Rationelle à l'usage des Elèves des Facultés des Sciences. By G. BOULIGAND. Tome I. *Avec un choix de problèmes proposés à la licence et à l'agrégation et rédigés.* Par M. J. DOLLON. Pp. viii + 282. 25 frs. 1925. (Vuibert, Boulevard Saint-Germain, 63 Paris, 5e.)

Arithmetic. Parts I. and II. By C. V. DURELL and R. C. FAWDRY. Pp. xiv + 156 + xx. 2s. 1925. (Bell.)

Notes on James Gray. Pp. 10. By C. TWEEDIE. Reprinted from the *Proc. of the Edinburgh Mathematical Society.* 1924.

Test Papers in Arithmetic, with Answers. By A. E. DONKIN. Pp. iv + 119 2s. 6d. 1925. (Pitman.)

A Shorter Geometry. By W. G. BORCHARDT and A. D. PERROTT. Pp. xi + 258 + xi. 4s. 1925. (Bell.)

Elementary Geometry. By C. V. DURELL. Pp. ix + 312. 4s. 6d. 1925. (Bell & Sons.)

Les Principes de la Théorie des Probabilités. Tome I. Fasc. i. *Principes et Formules du Calcul des Probabilités.* By E. BOREL. Edited by R. LAGRANGE. Pp. viii + 159. 18 fr. Tome II. Fasc. iii. *Mécanique Statistique Classique.* Pp. 148. 18 fr. 1925. (Gauthier-Villars.)

Introduction Géométrique à la Mécanique Rationelle. By C. CAILLER. Edited by H. FEHR and R. WAVRE. Pp. xii + 627. 60 fr. 1925. (Gauthier-Villars.)

Mémorial des Sciences Mathématiques. Fasc. II. *Fonctions entières et fonctions méromorphes d'une variable.* Pp. 57. By G. VALIRON. Fasc III. *Sur les Fonctions hypergéométriques de plusieurs variables, les Polynomes d'Hermite et autres fonctions sphériques dans l'hyperespace.* Pp. 72. By P. APPELL. 192. 10 fr. each. 1925. (Gauthier-Villars.)

Northumberland Standardised Tests. (1925 Series.) Prepared by Prof. CYRIL BURT. I. *Arithmetic,* pp. 8. II. *English,* pp. 19. *Manual of Instructions,* with Answers, pp. 22. The Set, 1s. 1925. (Univ. of London Press.)

A Treatise on Light. By R. A. HOUSTOUN. Pp. xi + 486. Fourth Edition. 12s. 6d. net. 1925. (Longmans, Green.)

Summation of Series. Collected by L. B. W. JOLLEY. Pp. x + 231. 13s. 6d. net. 1925. (Chapman, Hall.)

Analytical Geometry of Conic Sections and Elementary Solid Figures. By A. BARRIE GRIEVE. Pp. xv + 314 + xiv. 9s. 1925. (Bell.)

Einführung in die Analytische Geometrie. By A. SCHOENFLIES. Pp. x + 304. 15 Goldmark. 1925. (Springer, Berlin.)

Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität. (Teubner, Leipzig.)

April, 1925.

Ueber die Geometrie von Laguerre. III. *Beiträge zur Flachentheorie.* Pp. 1-12. W. BLASCHKE. *Berichtigung zu der Abhandlung "Zur Behandlung geschlossener Extremalen in der Variations-rechnung."* Pp. 13-14. J. RADON. *Abstrakte kontinuierliche Gruppen.* Pp. 15-32. O. SCHREIER. *Zur Theorie der diophantischen Approximationen.* II. *Der Real- und Imaginarteil von $\sigma(x; a)$* Pp. 33-46. H. BEHNKE. *Theorie der Zopfe.* Pp. 47-72. *Ueber Carathéodorys und Minkowskis Verallgemeinerungen des Längenbegriffs.* Pp. 73-116. T. ESTERMANN.

American Journal of Mathematics. (Johns Hopkins Press, Baltimore).

April, 1925.

Some Properties of the Exponential Mean. Pp. 71-90. J. DALE. *The Eliminant of a Net of Curves.* Pp. 91-98. F. MORLEY. *On an Equation of Planar Motion.* Pp. 98-100. F. MORLEY. *An Extension of the Problem of the Elastic Bar.* Pp. 101-120. H. T. DAVIS. *On the Sylow Subgroups of the Symmetric and Alternating Groups.* Pp. 120-124. L. WEISNER. *On certain Theorems regarding Summable Series and their Application to the Double and Triple Fourier's Series.* Pp. 125-139. G. M. MERRIMAN. *On the Power Characters of Units in a Cyclotomic Field.* Pp. 140-147. H. S. VANDIVER.

The American Mathematical Monthly. (Lancaster, Pa.)

May, 1925.

Is the Universe Finite? Pp. 213-223. A. HENDERSON. *The Euler Differential Equation of Infinite Order.* Pp. 223-233. H. T. DAVIS. *The Generalised Kronecker Symbol and its Application to the Theory of Determinants.* Pp. 233-241. F. D. MURNAGHAN. *The Circular Cubic on Twenty-one Points of a Triangle.* Pp. 241-247. T. W. MOORE and J. H. NEELLEY. *Note on certain practical problems which require the use of extended mathematical functions.* Pp. 247-248. C. E. VAN ORSTRAND. *On the occasional Need of very accurate Logarithms.* Pp. 249. H. BATEMAN. *Solution of the General Biquadratic.* Pp. 250-251. E. J. OGLESBY. *Note on the Introduction of Integral Calculus into a College Course in Solid Geometry.* Pp. 252-253. J. P. BALLANTINE. *Two new Arctangent Relations for π.* Pp. 253-255. A. A. BENNETT.

Annals of Mathematics. (Princeton University Press, N.J.)

March, 1925.

Non-monoidal Involutions which contain a Web of Invariant Monoids. Pp. 165-172. V. SNYDER. *Definite Integrals containing a Parameter.* Pp. 173-180. R. L. JEFFERY. *Integro-differential Expressions invariant under Volterra's Group of Transformations.* Pp. 181-201. A. D. MICHAL. *New Proofs of two well-known Theorems on Quadratic Forms.* Pp. 202-204. J. F. RITT. *Some Relations between Compound Determinants.* Pp. 205-211. W. H. METZLER. *A Contribution to the Theory of Interpolation.* Pp. 212-216. N. WIENER. *A Property of Cyclotomic Integers and its relation to Fermat's Last Theorem.* (Second Paper.) Pp. 217-232. H. S. VANDIVER. *On an Infinite System of Non-Abelian Groups of order nm^n.* Pp. 233-238. W. E. EDINGTON. *Conformal and Geodesic Mapping.* Pp. 239-246. A. BRAMLEY.

Bollettino della Unione Matematica Italiana. (Zanichelli, Bologna.)

June, 1925.

Applicabilità proiettiva di due superficie. Pp. 97-99. G. FUBINI. *Sulle funzioni quasi-analitiche di variabile reale.* Pp. 100-103. G. BELARDINELLI. *I concetti di limite e di integrale in un allievo di Cavalieri.* Pp. 104-107. A. AGOSTINI. *Sulle successioni uniformemente convergenti.* Pp. 107-109. G. VITALI. *Studio del moto incipiente di una trave pesante appoggiata su di un piano orizzontale scabro e sollicitata ad un estremo.* Pp. 109-113. G. BISCONCINI.

Bulletin of the American Mathematical Society. (Lancaster, Pa.)

March-April, 1925.

Remarks on the Foundations of Geometry. Pp. 121-141. O. VEBLEN. *Ricci's Coefficients of Rotation.* Pp. 142-145. H. LEVY. *Sur les valeurs asymptotiques des Coefficients de Cotes.* Pp. 145-156. J. OUSPENSKY. *Functions with an essential Singularity.* Pp. 157-162. P. FRANKLIN. *The Calculus of Variations.* Pp. 163-172. L. TONELLI.

May-June, 1925.

Life Insurance as a Social Service and as a Mathematical Problem. Pp. 227-252. *Simplifications relating to a Proof of Sylow's Theorem.* Pp. 253-256. G. A. MILLER. *Reduction of Euler's Equations to a Canonical Form.* Pp. 257-262. J. H. TAYLOR. *Integro-differential Equations.* Pp. 263-265. I. A. BARNETT. *A New Type of Double Sextette closed under a Binary (3, 3) Correspondence.* Pp. 266-274. L. D. CUMMINGS.

Bulletin of the Calcutta Mathematical Society. (Calcutta Univ. Press.)

March, 1925.

On the Construction of Partial Differential Equations of the second order satisfying assigned Conditions. Pp. 123-126. H. N. DATTA. *On two Pairs of Factorable Continuants.* Pp. 127-138. S. CHAKRAVARTY. *On the Evaluation of a Class of Definite Integrals.* Pp. 139-158. A. N. SINGH. *On the Stability of Vortex Rings of Finite Circular Section in Incompreible Fluids.* Pp. 169-172. N. SEN. *On the Time in Hyperbolic Orbits.* Pp. 173-176. B. C. DAS.

Contribución al Estudio de las Ciencias físicas y Matemáticas. (Univ. de la Plata.)

March, 1925.

The Eugenics Review. (Macmillan.)

April-July, 1925.

Intermédiaire des Mathématiciens. (Gauthier-Villars.)

May-June, 1925.

Jahresbericht der Deutschen Mathematiker Vereinigung. (Teubner, Leipzig.)

Vol. 34, Heft. 1-4.

Die Topologie der Mannigfaltigkeiten Pp. 1-14. H. KNESER. *Ueber die Entwicklung der Differentialgeometrie.* Pp. 14-25. D. J. STRUIK. *Fastperiodische Funktionen.* Pp. 25-40. H. BOHR. *R. Sturm.* Pp. 41-51. W. LUDWIG. *L. Krüger.* Pp. 52-57. K. HAUSSMANN. *Beiträge zur Theorie der Kugelfunktionen und ihrer Verallgemeinerungen.* Pp. 57-64. L. KOSCHMIEDER. *Ueber den Vektorenbereich eines konvexen ebenen Bereiches.* Pp. 64-80. H. RADEMACHER. *Ueber Polynome, die in Einheitswurzeln beschränkt sind.* Pp. 80-86. K. GRANDJOT.

The Journal of the Indian Mathematical Society. (Varadachari, Madras.)

April, 1925.

On the Instability of the Pear-Shaped Figure of Equilibrium of a Rotating Mass of Homogeneous Liquid (*continued*). Pp. 25-48. S. R. U. SAVOOR. *A Test for the Convergency and Divergency of a Series of Positive Terms.* Pp. 17-21. T. VIJAYARAGHAVAN. *An Elementary Definite Integral.* Pp. 22-27. G. S. MAHAJANI.

Journal of the Mathematical Association of Japan for Secondary Education. (R. Iwama. Tôkyô Higher Normal School for Girls.)

Feb.-April, 1925.

The Mathematics Teacher. (Yonkers, N.Y.)

May, 1925.

Mathematics in the Junior High School. Pp. 257-283. J. R. CLARK. *Mathematics and English.* Pp. 284-295. L. A. MACDONALD. *The Value of the History of Greek Mathematics to Secondary Schools.* Pp. 296-297.

Proceedings of the Edinburgh Mathematical Society. (Bell & Sons.)

May, 1925.

The Irreducible Covariants belonging to the Concomitant Systems of Three Quadrics. Pp. 2-16. J. WILLIAMSON. *The Geometrical Interpretation of the Complete System of two Double Binary* (2, 1) *Forms.* Pp. 17-26. W. SADDLER. *Power Coordinates and the Bicircular Quartic.* Pp. 26-34. R. WILSON. *On Self-Adjoint Partial Differential Equations of the Second Order.* Pp. 35-38. E. T. COPSON. *On the Riemann P Equation with Integral Exponent Differences.* Pp. 39-47. W. L. FERRAR. *A Generalization and Simple Proof of Kronecker's Theorem concerning the Minors of a Compound Determinant.* Pp. 48-49. W. H. METZLER. *Particular Solutions of the Equation of Conduction of Heat in One Dimension.* Pp. 50-63. M. C. GRAY. *Some Axi-Symmetric Determinants with Integers for Elements.* Pp. 64-69. J. J. NASSAU. *Notes on James Gray.* Pp. 70-80. C. TWEEDIE.

Proceedings of the Physico-Mathematical Society of Japan. (Faculty of Science, Tôkyô Imperial University.)

October, 1924.

On the Mean Value of L-Functions. Pp. 114-126. Z. SUETSUNA.

Nov. 1924.

On an Algebraical Problem related to an Analytic Theorem of Carathéodory and Fejér and on an allied Theorem of Landau. Pp. 130-140. T. TAKAGI.

Dec. 1924.

Le Champ Gravifique dû à une Sphère massique hétérogène. Pp. 142-146. T. OKAYA.

Jan.-Feb. 1925.

On a Generalised Scale of Notations. Pp. 34-47. S. KAKEYA.

May, 1925.

On Probability. Pp. 96-109. C. JORDAN.

Revista Matemática Hispano-Americana. (Soc. Mat. Española, Madrid.)

May, 1925.

Vito Volterra. Publicaciones del Prof. Volterra. Pp. 117-130. F. L. DE NÓ. *Construcciones Métricas y Resolución de Triángulos Esféricos en Proyección Estereográfica.* Pp. 131-140. P. P. ADAM.

June, 1925.

Sobre la ecuación de Pell. Pp. 149-159. J. REY PASTOR. *Un ejercicio de Geometria.* Pp. 160-164. R. U. LANA. *La Identidad de Vandermonde y el Teorema de Holdich.* Pp. 165-166. A. K. GARCÍA.

School Science and Mathematics. (Mount Morris, Illinois.)

Feb. 1925.

Hurdle Tests in Algebra. Pp. 132-144. K. WENTZ. *Arithmetic in the Junior High School.* Pp. 171-178. L. W. COLWELL.

March, 1925.

Algebra in the Junior High School. Pp. 271-286. E. C. HINKLE. *The Mathematics involved in solving High School Problems.* Pp. 292-299. G. W. REAGAN.

April, 1925.

Arithmetic in the Junior High School. Pp. 363-370. L. W. COLWELL. *The Three-Step Method of teaching Geometry.* Pp. 409-411. F. L. ABBOTT. *Graphical Methods and Lines of Force.* Pp. 412-415. R. C. COLWELL.

May, 1925.

The Teaching of Positive and Negative Numbers. Pp. 507-514. H. C. CHRISTOFFERSON. *Speed and Scholarship v. Arithmetical Accuracy.* Pp. 522-524. W. W. LUDEMAN.

June, 1925.

The proper Conception of a Normal Force. Pp. 578-589. R. T. CHASE. *Geometry in the Junior High School.* Pp. 611-617. J. J. JOHNSON.

Sitzungsberichte der Berliner Mathematischen Gesellschaft. (Kaestner, Göttingen.)

1923.

Über eine Fehlergesetzen gemeinsame Eigenschaft. Pp. 21-32. L. v. BORTKIEWICZ. *Zum Minkowskischen Sätze über lineare Formen.* E. JACOBSTHAL. *Über eine mit den Flächen zweiten Grades zusammenhängende zweifach-unendliche Schar aufeinander abwickelbarer algebraischer Flächen.* Pp. 42-63. H. JONAS. *Über die Variationsbreite ein Beobachtungsreihe.* Pp. 3-8. R. v. MISES. *Über eine Klasse von Mittelbildungen mit Anwendungen auf die Determinantentheorie.* Pp. 9-20. J. SCHUR. *Über eine Extremalaufgabe aus der Theorie der schlichten Abbildungen.* Pp. 38-47. G. SZEGÖ. *Die "Wurzel" aus einer homogenen linearen Substitution.* P. 48. C. WELTZIEN.

XXIII. Jahrgang.

Begründung der Theorie der linearen Integralgleichungen mit symmetrischen Kern mittels Variationsrechnung. Pp. 3-13. A. HAMMERSTEIN. *Beitrag zum Kräftruckproblem.* Pp. 14-45. H. REISSNER. *Zur Theorie der Luftkräfte.* Pp. 46-49. R. FUCHS. *Über eine Eigenschaft der Exponentialreihe.* Pp. 50-64. G. SZEGÖ.

XXIV. I.

Ueber eine Kroneckersche Konvergenzbedingung. Pp. 3-5. K. KNOPP. *Ueber einen Riemannschen Satz aus der Lehre von der konformen Abbildung.* Pp. 6-9. L. BIEBERBACH. *Eine bisher unbekannte Rede von Leonhard Euler.* Pp. 10-13. P. SCHAFHEITLIN. *Nach auf P. Schafheitlin.* Pp. 14-20. S. JOLLES. *Elektron und Kern als Punktladungen'.* Pp. 21-28. H. REISSNER. *Axiomatische Begrundung der geordneten Mengen.* Pp. 29-37. A. FRAENKEL. *Nachruf auf G. Wallenberg.* Pp. 3841. E. HAENTZSCHEL.

Sphinx-Oedipe. (A. Gérardin, Nancy.)

Jan.-May, 1925.

Unterrichtsblätter für Mathematik und Naturwissenschaften. (Salle, Berlin.)

Nos. 11 and 12, 1924.

Die Vorzeichen in der Geometrie. Pp. 126-130. C. THAER. *Zum Schluss von n auf n+1.* Pp. 130-132. W. LÖREY.

No. 1, 1925.

Stellung und Bedeutung der mathematisch-naturwissenschaftlichen Fächer in den höheren Schulen Würtembergs. Pp. 11-14. E. LÖFFLER.

No. 2, 1925.

Die Relativitätstheorie in der Schule. Pp. 27-28. A. KOPFF. *Wider alte Vorurteile der Infinitesimalrechnung gegenüber.* Pp. 28-32. A. HARNACK.

BELL'S LATEST BOOKS

Arithmetic. By C. V. DURELL, M.A., Senior Mathematical Master, Winchester College, and R. C. FAWDRY, M.A., Head of the Military and Engineering Side, Clifton College. In three parts. Parts I. and II., 2s., or separately, Part I., 10d.; Part II., 1s. 2d.; Part III., in preparation.

Based on modern methods and contains a large selection of practical, up-to-date examples. The authors' endeavour has been to introduce the pupil to the quickest method of solving any given type of example when he first meets with it. Revision Papers afford practice in rapid computation, while there are also sets of Problem Papers calling for a certain amount of ingenuity in their solution.

Elementary Geometry. By C. V. DURELL, M.A. 4s. 6d. Also in three parts, 2s. each.

In this Book Mr. Durell has adopted all the recommendations contained in the recent Report issued by the A.M.A., and in particular has followed the sequence of propositions which is the central feature of that report. There is an ample collection of numerical applications and easy riders.

A School Mechanics. By C. V. DURELL, M.A. Part I., 3s.; Parts II. and III., just out, 3s. each.

Parts I. and II. together cover the ground of the School Certificate and similar examinations; Part III. that of the Higher School Certificate.

"Mr. Durell's school books in other branches of mathematics are known to teachers, and we cordially recommend to them this latest. It is fully up to the author's own high standard."
Education Outlook.

A School Geometry on "New Sequence" Lines. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 4s. 6d. Also Books I.-III., 2s. 6d.; I.-V., 4s.

A systematic treatment of Geometry (including "solid") conforming to the recommendations of the A.M.A. Report. The different parts of the subject fall into Sections or Books; and in each Book the Theorems are arranged first, and are succeeded by the Problems (or Constructions). Exercises are numerous. Constructions are made practical.

A Shorter Geometry. By W. G. BORCHARDT, M.A., B.Sc., and Rev. A. D. PERROTT, M.A. 4s. Also in two parts, 2s. 6d. each.

A concise geometry on "new sequence" lines. The exercises, of which there are a large number, consist of numerical and construction examples, followed by ordinary riders.

Exercises in Geometry. By V. LE NEVE FOSTER, M.A. Part I. now ready, 2s. Answers, 9d. net.

Analytical Geometry of Conic Sections and Elementary Solid Figures. By A. BARRIE GRIEVE, M.A., D.Sc. 9s.

This new volume in "Bell's Mathematical Series" has been prepared primarily for Pass and Engineering Students, and for more advanced pupils in Secondary Schools. The first part is devoted to setting out in an easy and attractive way the easier properties of Conic Sections; the second gives an introduction to Solid Geometry, and includes a simple discussion of Quadric Surfaces, referred to their Principal Axes.

G. BELL & SONS, LTD.
PORTUGAL STREET, LONDON. W.C. 2.

THE MATHEMATICAL ASSOCIATION.

(An Association of Teachers and Students of Elementary Mathematics.)

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and an ornament thereunto."—BACON.

President:

Professor G. H. HARDY, M.A., F.R.S.

Vice-Presidents:

Prof. G. H. BRYAN, Sc.D., F.R.S.
Prof. A. R. FORSYTH, Sc.D., LL.D., F.R.S.
Prof. R. W. GENESE, M.A.
Sir GEORGE GREENHILL, M.A., F.R.S.
Sir T. L. HEATH, K.C.B., K.C.V.O., D.Sc., F.R.S.
Prof. E. W. HOBSON, Sc.D., F.R.S.
A. LODGE, M.A.
Prof. T. P. NUNN, M.A., D.Sc.
A. W. SIDDONS, M.A.
Prof. H. H. TURNER, D.Sc., D.C.L., F.R.S.
Prof. A. N. WHITEHEAD, M.A., Sc.D., F.R.S.
Prof. E. T. WHITTAKER, M.A., Sc.D., F.R.S.
Rev. Canon J. M. WILSON, D.D.

Hon. Treasurer:

F. W. HILL, M.A., 9 Avenue Crescent, Acton, London, W. 3.

Hon. Secretaries:

C. PENDLEBURY, M.A., 39 Burlington Road, Chiswick, London, W. 4.
Miss M. PUNNETT, B.A., The London Day Training College, Southampton Row, London, W.C. 1.

Hon. Secretary of the General Teaching Committee:

R. M. WRIGHT, B.A., Second Master's House, Winchester College, Winchester.

Hon. Secretary of the Examinations Sub-Committee:

W. J. DOBBS, M.A., 12 Colinette Rd., Putney, S.W. 15.

Editor of *The Mathematical Gazette*:

W. J. GREENSTREET, M.A., The Woodlands, Burghfield Common, Reading, Berks.

Hon. Librarian:

Prof. E. H. NEVILLE, M.A., B.Sc., 160 Castle Hill, Reading.

Other Members of the Council:

Prof. S. BRODETSKY, Ph.D., M.A., B.Sc.
A. DAKIN, M.A., B.Sc.
Miss M. J. GRIFFITH.
N. M. GIBBINS, M.A.
F. G. HALL, B.A.
H. K. MARSDEN, M.A.
Prof. W. P. MILNE, M.A., D.Sc.
Prof. W. M. ROBERTS, M.A.
W. F. SHEPPARD, Sc.D., LL.M.
C. O. TUCKEY, M.A.
C. E. WILLIAMS, M.A.

THE MATHEMATICAL ASSOCIATION, which was founded in 1871, as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and has exerted an important influence on methods of examination.

The Annual Meeting of the Association is held in January. Other Meetings are held when desired. At these Meetings papers on elementary mathematics are read and discussed.

Branches of the Association have been formed in London, Bangor, Yorkshire, Bristol, Manchester, Cardiff, Sydney (New South Wales), and Queensland (Brisbane). Further information concerning these branches can be obtained from the Honorary Secretaries of the Association.

"The Mathematical Gazette" (published by Messrs. G. BELL & SONS, LTD.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each. The *Gazette* contains—

(1) ARTICLES, mainly on subjects within the scope of elementary mathematics;

(2) NOTES, generally with reference to shorter and more elegant methods than those in current text-books;

(3) REVIEWS, written by men of eminence in the subject of which they treat. They deal with the more important English and Foreign publications, and their aim, where possible, is to dwell on the general development of the subject, as well as upon the part played therein by the book under notice;

(4) QUERIES AND ANSWERS, on mathematical topics of a general character.